

# Mathematics for Computer Graphics

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# Recommended Reading

- John Vince 2005. Mathematics for Computer Graphics Springer-Verlag London
- Comninos, P. 2005. Mathematical and Computer Programming Techniques for Computer Graphics (Hardcover). Springer.
- Harris J.W. and Stocker H. 1998 Handbook of Mathematics and Computational Science. Springer.

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- Lipschutz, S. 1982. Schaum's Outline of Essential Computer Mathematics (Paperback). Schaum's Outline Series.

# Syllabus

- Algebra
- Number Theory
- Boolean Algebra
- Circuit Diagrams
- Trigonometry
- Vectors
- Matrices
- Analytic Geometry
- 2D Transformations
- Set Theory

# Algebra

- One fundamental concept of algebra is the idea of giving a name to an unknown quantity.
- Rene Descartes (1596-1650) formalized the idea of using letters from the beginning of the alphabet ( $a, b, c$ , etc.) to represent arbitrary numbers, and letters at the end of the alphabet ( $p, q, r, s, t, \dots x, y, z$ ) to identify variables representing quantities such as pressure ( $p$ ), temperature ( $t$ ), and coordinates ( $x, y, z$ ).

# Algebra

- With the aid of the arithmetic operators we can develop expressions that describe the behavior of a specific computation.
- For example  $ax + by - d = 0$  represents a straight line.
- The variables  $x$  and  $y$  are the coordinates of any point on the line and the values of  $a, b, d$  determine the position and orientation of the line.
- There is an implied multiplication between  $ax$  and  $by$  which would be expressed as  $a * x$  and  $b * y$  using a programming language.
- The  $=$  sign permits the line equation to be expressed as a self-evident statement
- Such a statement implies that the expressions on the left-hand and right-hand sides of the  $=$  sign are **equal** or **balanced**.

# Algebraic Expressions

- Algebraic expressions also contain a wide variety of other notation, such as

$\sqrt{x}$  square root of  $x$ ;

$x^n$   $x$  to the power on  $n$

$\sin \alpha$  sine of  $\alpha$

- For example  $ax + by - d = 0$  represents a straight line.
- Parentheses are used to isolate part of an expression in order to select a sub-expression that is manipulated in a particular way. For example, the parentheses in  $c(a + b) + d$  ensure that the variables  $a$  and  $b$  are added together before being multiplied by  $c$  and finally added to  $d$ .

# Algebraic Laws

- There are three basic laws for manipulating algebraic expressions:
  - associative
  - commutative
  - distributive
- In the following descriptions, the term binary operation represents the arithmetic operations  $+$ ,  $-$  or  $\times$ , which are always associated with a pair of numbers or variables.



# Associative Law

- The *associative* law in algebra states that when three or more elements are linked together through a binary operation, the result is independent of how each pair of elements is grouped.

- Addition:

$$a + (b + c) = (a + b) + c$$

$$5 + (2 + 7) = (5 + 2) + 7$$

- Multiplication:

$$a \times (b \times c) = (a \times b) \times c$$

$$5 \times (2 \times 7) = (5 \times 2) \times 7$$

- Subtraction is not associative:

$$a - (b - c) \neq (a - b) - c$$

$$9 - (7 - 3) \neq (9 - 7) - 3$$

- What about the division, is it associative?

# Commutative Law

- The *commutative* law in algebra states that when two elements are linked through some binary operation, the result is independent of the order of the elements.

- Addition:

$$a + b = b + a$$

$$(2 + 7) = (7 + 2)$$

- Multiplication:

$$a \times b = b \times a$$

$$5 \times 9 = 9 \times 5$$

- Subtraction is not commutative:

$$a - b \neq b - a$$

$$15 - 6 \neq 6 - 15$$

- What about the division, is it commutative?

# Distributive Law

- The *distributive* law in algebra describes an operation which when performed on a combination of elements is the same as performing the operation on the individual elements.

- Multiplication over addition holds:

$$a \times (b + c) = (a \times b) + (a \times c) \qquad 5 \times (2 + 7) = (5 \times 2) + 5 \times 7$$

- Multiplication over subtraction holds:

$$a \times (b - c) = (a \times b) - (a \times c) \qquad 5 \times (7 - 4) = (5 \times 7) - 5 \times 4$$

- Addition over multiplication does not hold:

$$a + (b \times c) \neq (a + b) \times (a + c) \qquad 5 + (8 \times 2) \neq (5 + 8) \times (5 + 2)$$

# Powers(1)

- Indices (or powers, or exponents) are very useful in mathematics. Indices are a convenient way of writing multiplications that have many repeated terms.

- Example  $7^3$  means *multiply 7 by itself 3 times*

$$7^3 = 7 \times 7 \times 7$$

$$x^n = \underbrace{x \times x \times \dots \times x}_{n \text{ times}}$$

- Example  $6^{-4}$  means *multiply  $\frac{1}{6}$  by itself 4 times*

$$6^{-3} = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$$

$$x^{-n} = \underbrace{\frac{1}{x} \times \frac{1}{x} \times \dots \times \frac{1}{x}}_{n \text{ times}}$$

## Powers(2)

- Any number  $a$ ,  $a \neq 0$  raised to the power of 1 is  $a$   
$$a^1 = a$$

- Any number  $a$ ,  $a \neq 0$  raised to the power of -1 is  $\frac{1}{a}$   
$$a^{-1} = \frac{1}{a}$$

- Multiplying numbers with the same Base:  $x^m \times x^n = x^{m+n}$

- Dividing numbers with the same Base:  $\frac{x^m}{x^n} = x^{m-n}$

## Powers(2)

- Raising a power expression to a power:

$$a^{mn} = a^{m \times n} = a^{mn}$$

- Raising a power expression to a negative power:

$$a^{-m} = \frac{1}{a^m}$$

- Raising a product to a power:  $(a \times b)^m = a^m \times b^m$

- Raising a fraction to a power:  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

- There is no formula for adding variables raised to power:  $a^m + b^n$

## Powers(3)

- Square root is equivalent to raising a number to power  $\frac{1}{2}$

$$\sqrt{x} = x^{\frac{1}{2}} \quad \sqrt{49} = 49^{\frac{1}{2}} = 7$$

- Cubic root:

$$\sqrt[3]{x} = x^{\frac{1}{3}} \quad \sqrt[3]{216} = 216^{\frac{1}{3}} = 6$$

- n-root is equivalent to raising a number to power  $\frac{1}{n}$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

- Proot properties

- $\sqrt[n]{a \times b} = a^{\frac{1}{n}} \times b^{\frac{1}{n}}$
- $\sqrt[n]{a + b} \neq a^{\frac{1}{n}} + b^{\frac{1}{n}}$