Numbers

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Numbers

- **Natural Numbers:** The natural numbers 0, 1, 2, 3, 4, . are used for counting, ordering and labelling. We often use natural numbers to subscript a quantity to distinguish one element from another, e.g. x1, x2, x3, x4, etc.

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- **Integers:**Integers embrace negative numbers .. 2,10, 1, 2, 3, ..
- **Rational Numbers:**Rational or fractional numbers are numbers that can be represented as a fraction:
  For example
  $$0.25 = \frac{1}{4} \qquad \sqrt{16} = 4 = \frac{12}{4} \qquad 15 = \frac{15}{1} = \frac{30}{2}$$
  Some rational numbers can be stored accurately inside a computer, whilst many others can only be stored approximately. For example, 4/3 = 1.3333... produces an infinite sequence of threes and has to be truncated when stored as a binary number.

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- Therefore $x = 120/99 = 1.2121212121...$ is rational.

Number Systems

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- And any positive real number as

$$abcd.efg = a \times 10^3 + b \times 10^2 + c \times 10^1 + d \times 10^0 + e \times 10^{-1} + f \times$$

where a, b, c, d, e, f, g are all ≤ 9

Octal to Decimal

- Ten is used only because we have ten fingers. If we had 8 fingers we would define an octal positive integer as.

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- table:

0	1	2	3	4	5	6	7
10	11	12	13	14	15	16	17
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- $10_8 = 8_{10}$ $\quad 12_8 = 10_{10}$ $\quad 20_8 = 16_{10}$

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- e.g.

$$\begin{aligned} 11010_2 x &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 \\ &= 26_{10} \end{aligned} \quad (2)$$

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- $$10100_2 = 20_{10}$$

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- $$12021_3 = 1 \times 3_4 + 2 \times 3_3 + 0 \times 3^2 + 2 \times 3^1 + 1 \times 3^0 = 142_{10}$$

Hexadecimal Numbers (base 16)

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$$\begin{aligned} 2E6_{16} &= 2 \times 16^2 + E \times 16^1 + 6 \times 16^0 \\ &= 2 \times 16^2 + 14 \times 16^1 + 6 \times 16^0 \\ &= 742_10 \end{aligned}$$