Trigonometry

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- The perimeter of a circle equals $2\pi r$, therefore 2π radians correspond to one complete rotation.
  360^o correspond to 2π radians, therefore 1 radian corresponds to $180/\pi$ degree, approximately 57.3^o.

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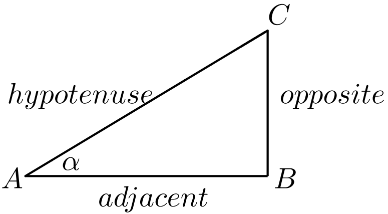
- 

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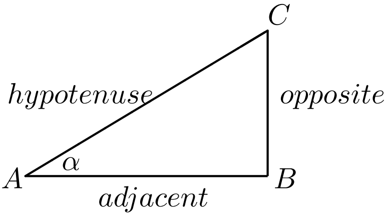
- 

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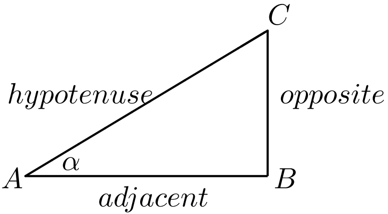
- 

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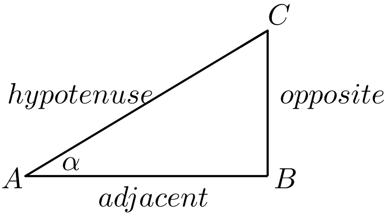
- 

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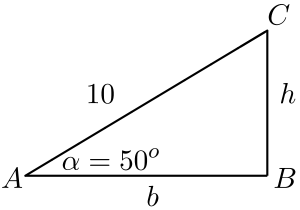
- $$\tan(\alpha) = \frac{opposite}{adjacent} \qquad \text{cotan}(\alpha) = \frac{1}{\tan(\alpha)}$$

Example

- Given a triangle where the hypotenuse and one angle are known. The other sides are calculated as follows.



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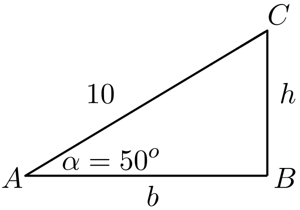

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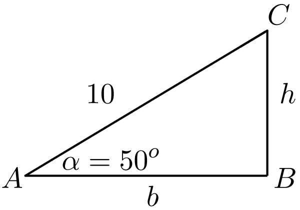

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$$h = 7.66 \quad (1)$$

-

$$\frac{b}{10} = \cos(\alpha) \Rightarrow b = 10\cos(50_o)$$
$$b = 6.4279 \quad (2)$$

Inverse trigonometric ratios

- The $\sin$, $\cos$ and $\tan$ functions convert angles into ratios, and the inverse functions $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ convert ratios into angles.

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$$
\begin{array}{lllll}
\sin^{-1}(x) = \theta & \text{where} & -\pi/2 \leq \theta \leq \pi/2 & \text{and} & \sin(\theta) = x \\
\cos^{-1}(x) = \theta & \text{where} & 0 \leq \theta \leq \pi & \text{and} & \cos(\theta) = x \\
\tan^{-1}(x) = \theta & \text{where} & -\pi/2 \leq \theta \leq \pi/2 & \text{and} & \tan(\theta) = x
\end{array}
$$

Trigonometric relationships

- There are intimate relationships between the $\sin$ and $\cos$ definitions and are formally related by

$$\cos(\beta) = \sin(90^o - \beta)$$
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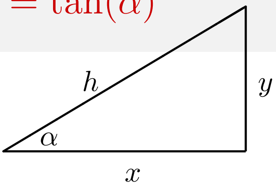
- The Theorem of Pythagoras can be used to derive other formulae such as:

  - $$\frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha) \quad (3)$$
  - $$\sin^2(\alpha) + \cos^2(\alpha) = 1 \quad (4)$$
  - $$1 + \tan_2(\alpha) = \sec^2(\alpha) \quad (5)$$
  - $$1 + \cot^2(\alpha) = \mathrm{cosec}^2(\alpha) \quad (6)$$

$\frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha)$

- 

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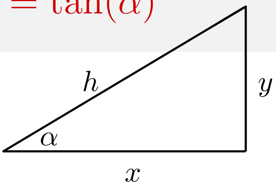
- 

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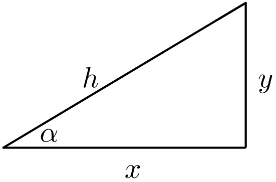
$$\sin(\alpha) = \frac{y}{h}$$
$$\cos(\alpha) = \frac{x}{h}$$
$$\Downarrow$$
$$\frac{\sin(\alpha)}{\cos(\alpha)} = \frac{y}{h}\frac{h}{x} = \frac{y}{x}$$
$$\Downarrow$$
$$\frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha)$$

$\sin^2(\alpha) + \cos^2(\alpha) = 1$

- 

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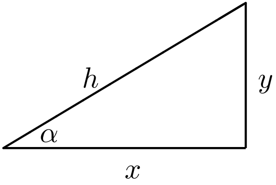
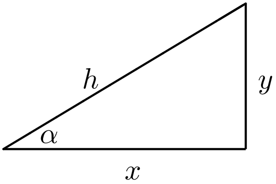
- 

- Using the theorem of Pythagoras:

$$x^2 + y^2 = h^2$$
$$\frac{y^2}{h^2} + \frac{x^2}{h^2} = 1$$
$$\Downarrow$$
$$\left(\frac{y}{h}\right)^2 + \left(\frac{x}{h}\right)^2 = 1$$
$$\Downarrow$$
$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$1 + \tan^2(\alpha) = \sec^2(\alpha)$

- 

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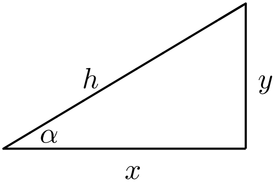
- 

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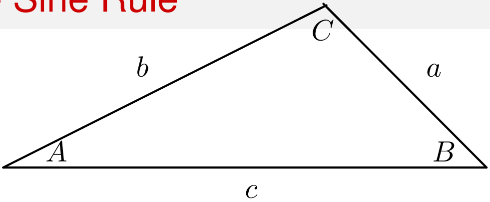
$$\begin{aligned} \cos^2(\theta) + \sin^2(\theta) &= 1 \\ \frac{\cos^2(\theta)}{\cos^2(\theta)} + \frac{\sin^2(\theta)}{\cos^2(\theta)} &= \frac{1}{\cos^2(\theta)} \\ &\Downarrow \\ 1 + \tan^2(\alpha) &= \sec^2(\alpha) \end{aligned}$$

The Sine Rule



$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{b}{\sin(B)}$$

- **Example:** $A = 50^o$, $B = 30^o$, $a = 10$, find b.

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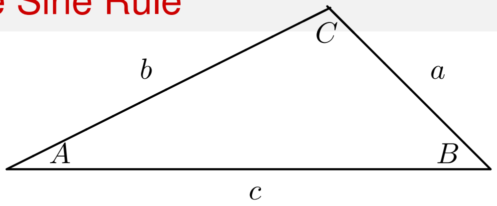

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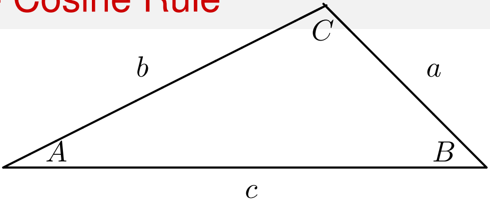
$$\frac{b}{\sin(30)} = \frac{10}{\sin(50)}$$
$$b = \frac{10\sin(30)}{\sin(50)}$$
$$b = 6.5274$$

The Cosine Rule



- $$a^2 = b^2 + c^2 - 2bc\cos(A)$$
  $$b^2 = c^2 + a^2 - 2ca\cos(B)$$
  $$c^2 = a^2 + b^2 - 2ab\cos(C)$$

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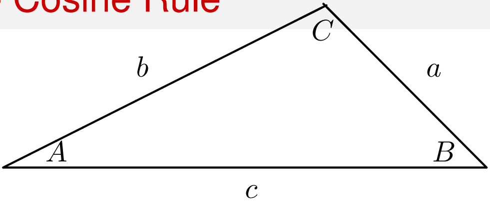

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  $$b^2 = c^2 + a^2 - 2ca\cos(B)$$
  $$c^2 = a^2 + b^2 - 2ab\cos(C)$$

- $$a = b\cos(C) + c\cos(B)$$
  $$b = c\cos(A) + a\cos(C)$$
  $$c = a\cos(B) + b\cos(A)$$

Compound angles

- Two sets of compound trigonometric relationships show how to add and subtract two different angles
- The following are some of the most common relationships

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \\
\sin(\alpha-\beta) &= \sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta) \\
\\
\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta) \\
\cos(\alpha-\beta) &= \cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta) \\
\\
\tan(\alpha+\beta) &= \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)} \\
\tan(\alpha-\beta) &= \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}
\end{aligned} \quad (8)$$

Compound angles

- Using the previous relationships, we can show the relationships
- for multiples of the same angle.

$$\begin{aligned}
\sin(2\alpha) &= 2\sin(\alpha)\cos(\alpha) \\
\cos(2\alpha) &= \cos^2(\alpha) - \sin^2(\alpha) \\
\cos(2\alpha) &= 2\cos^2(\alpha) - 1 \\
\cos(2\alpha) &= 1 - 2\sin^2(\alpha)
\end{aligned}$$

$$\begin{aligned}
\sin(3\alpha) &= 3\sin(\alpha) - 4\sin^3(\alpha) \\
\cos(3\alpha) &= 4\cos^3(\alpha) - 3\cos^2(\alpha)
\end{aligned}$$

$$\begin{aligned}
\sin^2(\alpha) &= \frac{1}{2}(1 - \cos(2\beta)) \\
\cos^2(\alpha) &= \frac{1}{2}(1 + \cos(2\beta))
\end{aligned}$$