**ASSESSMENTS OFFICE**

## BA (HONS) COMPUTER VISUALISATION AND ANIMATION

Answer **FOUR** questions

**Year:** 1st

**Date:** 08/06/04

**Time:** 09.30-12.30

ALL questions carry equal marks

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## MATHEMATICS FOR COMPUTER GRAPHICS 1
## BACVA1F.03.3

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## Question 1 (4 Parts)

**1.1.** Our decimal number system is a positional number system using the base 10. Give a general expression that describes how a number for any base. Illustrate your answer using the base 2.

**[5 marks]**

**1.2.** Define and give one example of each of the following

a: A natural number.
B: A integer.
C: An irrational number.
D: A real number.
E: A transcendental number. **[5 marks]**

**1.3.** Calculate the binary value of X for the following expressions:

a: $X_{10} = 10011_2 + 105_6 - 23_4$
b: $X_2 = \sqrt{1011_{10} + 100_2 + 100_3}$
c: $X_8 = (110_2 + 20_4) \times (32_5 - 100_3)$ **[8 marks]**

**1.4** Let a+b.i be a complex number.
Then *a-b.i* is called the *conjugate* of this complex number (i.e. the same real part but the sign of the imaginary part is reversed.)
Similarly, a+b.i is the conjugate of a-b.i.
Show what this means by way of a graphical representation.
Prove that the square of a complex number and the square of its conjugate are also conjugates of each other.

**[7 marks]**

## Question 2 (4 Parts)

**2.1.** Sketch and annotate the graphs of the following functions over the range $0 \le x \le 2\pi$ radians.

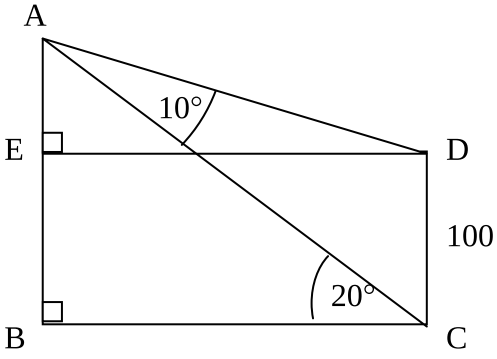
A: cos(2x)
b: 2sin(x)
c: cos(x) + 2
e: $\sin^2(x) - \cos^2(x)$ **[5 marks]**

**2.2.** Prove the following:

a: tan(x) = sin(x)/cos(x)
b: $\sin^2(x) + \cos^2(x) = 1$
c: $1 + \tan^2(x) = \sec^2(x)$ **[5 marks]**

**2.3.** In the following diagram, find the distance AB, given that:

CB = DE; EB = DC = 100; Angle EDA = 10°; Angle BCA = 20°




**[9 marks]**

**2.4.** Given that sin(A+B) = sinAcosB + cosAsinB, and cos(A+B) = cosAcosB - sinAsinB

prove that: $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

**[6 marks]**

## Question 3 (4 Parts)

**3.1.** Describe, with the aid of an example, the following:

a: Vector addition
b: The magnitude of a 3D vector
c: A unit 2D vector
d: Cartesian vector components **[8 marks]**

**3.2.** Three vectors **a**, **b** and **c** are used to construct 2 other vectors **A** and **B** as follows:

**A** = |**b**|.**a** – **c** and **B** = |**c**|.**b** + **a**

Using the scalar (dot) product, calculate the angle between the vectors **A** and **B** where:

**a** = $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ **b** = $\begin{bmatrix} 5 \\ 12 \end{bmatrix}$ **c** = $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$ **[8 marks]**

**3.3.** If **u** = a**i** + b**j** + z**k** and **v** = d**i** + e**j** + f**k**, what is their vector (cross) product in terms of their cartesian components, and what does this product represent in terms of the vectors **u** and **v**?

**[4 marks]**

**3.4.** Describe how we might use the cross product to calculate the surface area of a polygon made up of triangles and/or parallelograms. Use a sketch if necessary.

**[5 marks]**

## Question 4 (3 Parts)

**4.1.** Describe, with the aid of graphs, how the linear function:

$$V = tV_1 + (1-t)V_2$$

:interpolates between the values $V_1$ and $V_2$. **[5 marks]**

**4.2.** Describe how the above linear interpolant can be developed to support quadratic interpolation. **[7 marks]**

**4.3.** A Bezier curve is constructed between the points (1,1) and (4,-4) and (3,6) as a control point. If the controlling parameter over the range of the curve is where $0 \le t \le 1$, calculate the points on the curve for the following values of *t*.

a: $t = 0$
b: $t = 0.25$
c: $t = 0.5$
d: $t = 1$ **[8 marks]**

**4.4.** State Pascal's Triangle for the first 5 rows, and hence or otherwise give the first 5 binomial expansions, i.e. for n = {0, 1,2 ,3, 4}.
**[5 marks]**

## Question 5 (3 Parts)

**5.1.** Describe the following terms:

a: transpose of a matrix
b: shearing matrix
c: rotation matrix
d: translating matrix **[8 marks]**

**5.2.** The co-ordinates of a 2D shape are to be scaled relative to the point (2,1) by a factor of 3 in the x-direction and 2 in the y-direction; they are then finally translated by 3 in the x-direction and 3 in the y-direction.

Derive the individual homogenous transformations for these actions, and concatenate them fully into a single homogenous matrix.
**[10 marks]**

**5.3.** Show, with the aid of an example, that matrix operations are not commutative
**[7 marks]**

## Question 6 (3 Parts)

**6.1.** Illustrate the meaning of the scalars a, b and d in the line equation ax +by – d = 0, and briefly explain how this equation can be used to partition space.

**[9 marks]**

**6.2.** What is the equation of the line passing through the points $P_{1} =$ (3, 5) and $P_{2} =$ (5, 6)? Write the answer in Hessian normal form, and give the X and Y axes-intercepts

**[8 marks]**

**6.3.** If the equation of a circle is $r^2 = x^2 + y^2$, where the radius *r* = 1, calculate, algebraically, the intersection points with the line $y = -\frac{x}{2} - \frac{1}{2}$

**[8 marks]**

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