ASSESSMENTS OFFICE

BA (HONS) COMPUTER VISUALISATION AND ANIMATION

Answer **FOUR** questions	**Level:**	C
	Date:	6 June 2005
Graph paper to be provided	**Time:**	09.30 – 12.30

ALL questions carry equal marks

MATHEMATICS FOR COMPUTER GRAPHICS 1
BACVAF.C.04.3

INSTRUCTIONS TO CANDIDATES

Calculators may be used for this exam

Question 1 (this question has five parts)

1.1. Calculate the value of X_n for the following, where the subscript represents the base of the number:

a: $X_{16} = (144_8)^2$
b: $X_2 = \sqrt[4]{1331_6}$
c: $X_5 = \cos(10_2 \times \pi) + 47_8$

[6 marks]

1.2. Describe how the $\sqrt{-1}$ rotates a complex number through 90°. Use a diagram to illustrate your answer.

[5 marks]

1.3. Define and and give one example of each of the following

a: An integer number.
b: A rational number.
c: A complex number.
d: A prime number.
e: A transcendental number.

[5 marks]

1.4. Our decimal number system is a positional number system using the base 10. Describe a number system using the base 5, showing how integer quantities are represented left of the decimal point for four four terms, and fractional quantities to the right for three three terms.
Give your answer in the form of a general expression complete with an necessary bounds.

[4 marks]

1.5. The formula for calculating the new amount p^1 after applying annual interest rate r to initial amount p is:

$$p^1 = p + \frac{r}{100} p = p(1 + \frac{r}{100})$$

Give a general formula for the compound interest earnt over k years.
To the nearest whole percentile, how much interest is needed to turn £5000 into *at least* £7000 over the course of 7 years?

[5 marks]

Question 2 (this question has three parts)

2.1 Sketch the graphs of the quadratic Bernstein polynomials, labelling each curve. Show how we might alter the curvature of the quadratic Bernstein interpolant curve between values V_1 and V_2 using the control value V_c, and determine the value of V_c (in terms of V_1 and V_2) where we obtain linear interpolation.

[10 marks]

2.2 A Bezier curve is constructed between the points (2,4) and (7,2), and (3,9) as a control point. If the controlling parameter over the range of the curve is where $0 \leq t \leq 1$, calculate the points on the curve for the following values of *t*.

a: $t = 0$
b: $t = 0.25$
c: $t = 0.5$
d: $t = 1$

[8 marks]

2.3 In the context of Bezier curves, describe the following terms, with the aid of a diagram:

A: convex hull
B: piecewise curves
C: the joining of curve segments with C^1 continuity.

[7 marks]

Question 3 (this question has three parts)

3.1 Illustrate the meaning of the scalars a, b and d in the line equation ax + by – d = 0, and briefly explain how this equation can be used to partition space.

[9 marks]

3.2 Given the triangle defined by the points P_1 = (2, -1), P_2 = (4, 4) and P_3 = (-1, 2), determine where the following points are in relation to it, using Hessian normal form to represent the triangle's edges:

P_{t1} = (0, 4)
P_{t2} = (2, 1)
P_{t3} = (1, 0)

[8 marks]

3.3 Describe the Hessian normal form of a plane equation, and brielfy describe how it, too, can be use to partition space.

[8 marks]

Question 4 (this question has three parts)

4.1 In the context of 2D geometric transformations describe and give an example of each of the following:

a.Identity matrix
b.Translation matrix
c.Scaling matrix
d:Homogeneous (Barycentric) co-ordinates
e:Concatenation of matrices

[10 marks]

4.2 Derive a single matrix to undertake the following collective 2D transformations. An object is scaled by a factor of 2 about the point (2,2), then translated by 3 in the x-direction and 2 in the y-direction.

[10 marks]

4.3 Show that matrix operations are non commutative.

[5 marks]

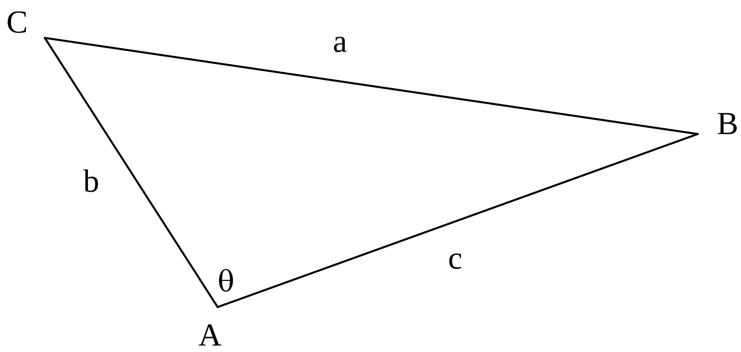
Question 5 (this question has four parts)

5.1 Given $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$, find $\cos(2\alpha)$ in terms of cosines only.

From this, find $\cos(3\alpha)$ in terms of cosines only.

[7 marks]

5.2 Using the following diagram, prove that $a^2 = b^2 + c^2 – 2bc.\cos(\theta)$



[7 marks]

5.3 Prove the following:

a: $\tan(x) = \sin(x)/\cos(x)$
b: $\sin^2(x) + \cos^2(x) = 1$
c: $1 + \cot^2(x) = \text{cosec}^2(x)$

[5 marks]

5.4 Given the following equation for solving quadratics:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

: find the any and all root(s) of the following equations:

a: $4x^2 +25x - 26$
b: $25x^2 - 20x + 4$
c: $7x^2 – 2x + 6$

[6 marks]

Question 6 (this question has three parts)

6.1 Explain briefly how we normalise a vector, and why it is useful to have unit vectors in scene calculations.

[4 marks]

6.2 Given $\mathbf{r} = a_r\mathbf{i} + b_r\mathbf{j} + c_r\mathbf{k}$ and $\mathbf{s} = a_s\mathbf{i} + b_s\mathbf{j} + c_s\mathbf{k}$ and $\mathbf{r}\bullet\mathbf{s} = |\mathbf{r}|\,|\mathbf{s}|\cos(\theta)$, where θ is the angle between **r** and **s**, show that $\mathbf{r}\bullet\mathbf{s} = a_ra_s + b_rb_s + c_rc_s$.

[5 marks]

Using the scalar product calculate the angle between $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{s} = 3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$

[4 marks]

6.3 If $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and $\mathbf{v} = d\mathbf{i} + e\mathbf{j} + f\mathbf{k}$, what is their vector (cross) product in terms of their cartesian components?

[3 marks]

If $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ and $\mathbf{v} = -\mathbf{i} + 3\mathbf{j} - 1\mathbf{k}$, calculate the new vector given by the vector $\mathbf{u}\times\mathbf{v}$.
Explain in detail the relationship between the area of a parallelogram and the vector product.
Use both of the above to calculate both the angles in the parallelogram that can be constructed if **u** and **v** constitute its edges.

[9 marks]

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