BA (HONS) COMPUTER VISUALISATION AND ANIMATION

Answer **FOUR** questions

Year: 1

Time: 9.30-12.30

Date: 5/6/2006

MATHEMATICS FOR COMPUTER GRAPHICS 1

Calculators may be used.

Graph paper will be provided.

1

1.1 Compute the value of X_b for the following where the subscript represents the base of the number.

a: $X_3 = 24_{10}$

b: $X_4 = 2_{10}\sqrt{100_5}$

c: $X_8 = \sqrt{1001_2 \times 100_2}$

d: $X_5 = 121_5 + 101_4$

e: $X_2 = \left(100_8\right)^{\frac{1}{2}}$ **[5 marks]**

1.2 Express the sum of the first five prime numbers as a binary number. **[5 marks]**

1.3 Define the meaning of the following and give an example of each.

a: a rational number
b: a scalar quantity
c: a quaternion
d: a complex number
e: a vector quantity **[5 marks]**

1.4 Illustrate how $\sqrt{-1}$ rotates a complex number through 90º. **[5 marks]**

1.5 Simplify the following complex numbers.

a: $(1+i2)+(2-i3)$

b: $(-1-i2)-(-4-i3)$

c: $(2+i4)\times(3+i5)$

d: $(1+i)^2$

e: $i^1+i^2+i^3+i^4$ **[5 marks]**

2

2.1 Prove the following identities.

a: $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

b: $\sin^2\alpha + \cos^2\alpha = 1$

c: $1 + \tan^2\alpha = \sec^2\alpha$ **[6 marks]**

2.2 Sketch and annotate the graphs of the following functions over the range $0 \le \alpha \le 2\pi$ radians.

a: $2\sin\alpha$

b: $\sin 3\alpha$

c: $\sin^2\alpha$ **[6 marks]**

2.3 Simplify the following expressions.

a: $\log\left(10^{2\sin 90^\circ}\right)$

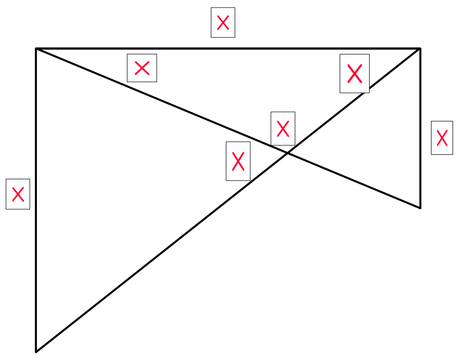
b: $(x-1)^3$

c: $(x+y)^2 - (x-y)^2$

[6 marks]

2.4 From the diagram (not to scale) calculate the angles $\alpha, \theta, \beta, \phi$.

[7 marks]



3

3.1 Given the vectors **a**, **b** and **c** compute the following vector equations.

$$\mathbf{a} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

a: $\mathbf{a}+\mathbf{b}+\mathbf{c}$
b: $\mathbf{b}-2\mathbf{a}$
c: $\mathbf{c}-\mathbf{b}-\mathbf{a}$
d: $\mathbf{b}\cdot\mathbf{c}$ **[4 marks]**

3.2 If $\mathbf{a}=4\mathbf{j}+3\mathbf{k}$ and $\mathbf{b}=2\mathbf{i}-3\mathbf{j}+5\mathbf{k}$ calculate the following.
a: $\|\mathbf{a}\|$
b: $\|\mathbf{b}\|$
c: $\|\mathbf{a}\|\mathbf{b}$ **[6 marks]**

3.3 Given $\mathbf{r}=a_r\mathbf{i}+b_r\mathbf{j}+c_r\mathbf{k}$ and $\mathbf{s}=a_s\mathbf{i}+b_s\mathbf{j}+c_s\mathbf{k}$ and $\mathbf{r}\cdot\mathbf{s}=\|\mathbf{r}\|\|\mathbf{s}\|\cos\alpha$, where α is the angle between $\mathbf{r}$ and $\mathbf{s}$, show that $\mathbf{r}\cdot\mathbf{s}=a_ra_s+b_rb_s+c_rc_s$.
[5 marks]

3.4 Using the scalar product calculate the angle between **r** and **s** where $\mathbf{r}=2\mathbf{i}+3\mathbf{j}+6\mathbf{k}$ and **s = i** + 4**j** – 2**k**. **[5 marks]**

3.5 Describe how the scalar product is useful in identifying back-facing polygons.
[5 marks]

4

4.1 Give the algebraic equivalent of the following matrices.

a: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

b: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

c: $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

d: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

[6 marks]

4.2 Describe the following 2D matrices and give an example of each.
a: a scaling matrix
b: a rotation matrix
c: a reflection matrix
d: a translation matrix. **[8 marks]**

4.3 Describe how homogeneous coordinates provide a mechanism for combining scaling and translation matrices. Illustrate your answer with an example.

[5 marks]

4.4 Construct a 2D matrix that reflects points about the line $x = 1$.**[6 marks]**

5

5.1 Describe how projective geometry can be used to create a perspective view of a 3D object, and derive the equations for computing the coordinates of points on a projection plane. **[10 marks]**

5.2 Given that an observer is located at the origin and is looking along the z axis, find the projection coordinates of the following 3D points $(10,10,30)$, $(20,20,40)$. The projection plane is 10 units from the observer and is orthogonal to the z axis.

[4 marks]

5.3 Describe how a pseudo fish-eye effect can be obtained and derive the associated projective equations. **[7 marks]**

5.4 Describe the role of the viewing frustum in the projection process.

[4 marks]

6

6.1 Describe the geometric meaning of a, b and c in the Cartesian form of the line equation $ax + by = c$ and illustrate your answer with an example.

[6 marks]

6.2 Show how line equations can be used to determine whether a point is inside or outside a convex 2D polygon. In your answer describe how it is also possible to detect whether a point is located on an edge or vertex.**[10 marks]**

6.3 Compute the shortest distance from the origin to a plane if its equation is $2x + 3y - 4z = 10$. **[4 marks]**

6.4 Describe how plane equations can be used to determine whether points are inside, outside, or located at an edge or vertex of a convex volume **[5 marks]**