Bournemouth University

BA (HONS) COMPUTER VISUALISATION AND ANIMATION

Answer **FOUR** questions

Year: 1

Time:

Date:

MATHEMATICS FOR COMPUTER GRAPHICS 1

Calculators may be used.

Graph paper will be provided.

1

1.1 Give the Algebraic equivalent of the following matrices

a: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

b: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1.5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

c: $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

d: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

[7 marks]

1.2 Describe the following 2D matrices and give an example of each

a: a scaling matrix
b: a rotation matrix
c: a reflection matrix
d: a translation matrix

[8 marks]

1.3 Derive a single matrix that contains the concatenation of the following 2D transformations: An object is scaled by a factor of 2 about the point P(2,2), then translated by 3 in the *x* direction and by 2 in *y* direction.

[10 marks]

2.

2.1 Given that $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$, prove the following identities.

a: $\cos(2\theta)=1-2\sin^2(\theta)$

b: $\cos(3\theta)=4\cos^3(\theta)-3\cos(\theta)$

c: $\cos^2(\theta)=\frac{1}{2}(1+\cos(2\theta))$

[6 marks]

2.2 Sketch and annotate the graphs of the following functions over the range $0<\alpha<2\pi$ radians.

a: $\cos(\alpha)$

b: $\cos(2\alpha)$

c: $\sin^2(\alpha)$

[6 marks]

2.3 Simplify the following expressions:

a: $\log\left(10^{(2\cos(180^o))}\right)$

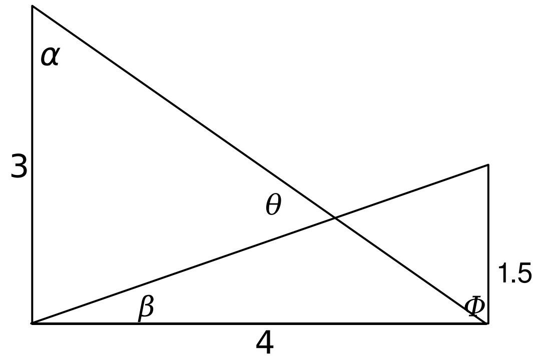
b: $(x-1)^3$

c: $(a-b)^2-(a+b)^2$

[6 marks]

2.4 From the diagram (not to scale) calculate the angles $\alpha, \theta, \beta, \phi$.

[7 marks]



3.

3.1 Give a general expression that describes how a number is represented in any base. Illustrate your answer using the base 2.

[5 marks]

3.2 Define and give one example of each of the following

a: a natural number
b: an integer number
c: an irrational number
d: a real number
e: a complex number

[5 marks]

3.3 Calculate the value of X for the following expressions:

a: $X_{10}=101011_2+32_4+105_6$

b: $X_2=\sqrt{10011_{10}+100_2+100_3}$

c: $X_8=\left(110_2+20_4\right).\left(32_5-100_3\right)$

[8 marks]

3.4 Simplify the following complex numbers

a: $(1-4i) + (3+4i)$
b: $(1-4i) \times (3+4i)$
c: $(a-bi) \times (a+bi)$ where a and b are real numbers
d: $i^2 + i^3 + i^4$
e: $i^{-1} + i^{-2} + i^{-3}$

[7 marks]

4

4.1 Illustrate the meaning of scalars a, b and c in the line equation $ax + by - c = 0$; and briefly explain how this equation can be used to partition space.

[9 marks]

4.2 What is the equation of the line passing through the points P(3,-2) and Q(-1,2)? Write the answer in Hessian normal form, and give the X and Y axes-intercepts.

[8 marks]

4.3 If the equation of a circle is $r^2 = x^2 + y^2$,where the radius is r=1, calculate, algebraically the intersection with the line y = -0.5x-0,5

[8 marks]

5

5.1 Describe, with the aid of an example, the following:

a: Vector addition

b: The magnitude of a 3D vector

c: A unit 2D vector

d: Cartesian vector components

[8 marks]

5.2. Three vectors ***a***, ***b*** and ***c*** are used to construct two other vectors **A** and **B** as follows:

$$\mathbf{A} = |\boldsymbol{b}|.\boldsymbol{a} - \boldsymbol{c} \text{ and } \mathbf{B} = |\boldsymbol{c}|.\boldsymbol{b} + \boldsymbol{a}$$

Using the scalar (dot) product, calculate the angle between the vectors **A** and **B** where:

$$\boldsymbol{a}=\begin{bmatrix}1\\3\end{bmatrix};\boldsymbol{b}=\begin{bmatrix}5\\10\end{bmatrix};\boldsymbol{c}=\begin{bmatrix}4\\-3\end{bmatrix}$$

[8 marks]

5.3. If $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + z\mathbf{k}$ and $\mathbf{v} = d\mathbf{i} + e\mathbf{j} + f\mathbf{k}$, what is their vector (cross) product in terms of their Cartesian components, and what does this product represent in terms of the vectors **u** and **v**?

[4 marks]

5.4. Describe how we might use the cross product to calculate the surface area of a polygon made up of triangles and/or parallelograms. Use a sketch if necessary.

[5 marks]

6

6.1 Describe, with the aid of graphs, how the linear function:
V = tV1 + (1-t)V2 interpolates between the values *V1* and V2.

[5 marks]

6.2. Describe how the above linear interpolant can be developed to support quadratic interpolation.

[7 marks]

4.3. A Bezier curve is constructed between the points (1,1) and (3,-3) and (3,6) as a control point. If the controlling parameter over the range of the curve is where $0 \leq t \leq 1$, calculate the points on the curve for the following values of t.

a: $t = 0$
b: $t = 0.25$
c: $t = 0.50$
d: $t = 0.75$

[8 marks]

4.4. State Pascal's Triangle for the first 5 rows, and hence or otherwise give the first 5 binomial expansions, i.e. for n = {0, 1, 2, 3, 4}.

[5 marks]