Bournemouth University

BA (HONS) COMPUTER VISUALISATION AND ANIMATION

Answer **FOUR** questions

Year: 1

Time:

Date:

MATHEMATICS FOR COMPUTER GRAPHICS 1

Calculators may be used.

Graph paper will be provided.

Question 1

1.1. Define and illustrate by Venn diagrams the following:
   a. union set operation
   b. difference set operation

[5 marks]

1.2. Define the following sets:
   a. {a,b,c,d} ∪ {2,3} =
   b. {2,1,5} ∪ {1,5,7} =
   c. {a,b,c} ∩ {2,3} =
   d. {2,1,5} ∩ {1,5,7} =
   e. {1,2,3,4,5,6} – {2,3,5,7,9,6} =
   d. {3,4} – {4,3,1} =

[7 marks]

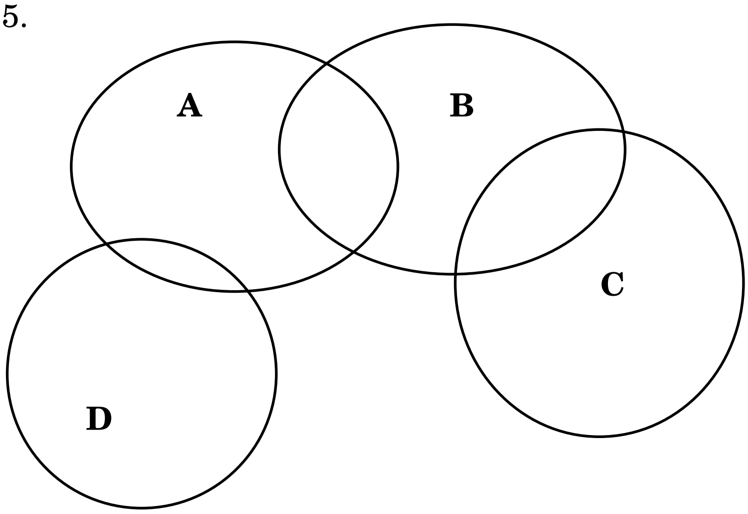
1.3. Let $S=\{x \mid x \subseteq \{1,2,3\}\}$. Make a list of all possible elements of S.

[5 marks]

1.4. What is the cardinality of the set {{1,2,3}, {4,5}, {7,6}} ?

[3 marks]

1.5.



(A∩B) - (C∪D)

   a. Mark in black the final result of the above operations.
   b. Cross out the redundant sets, which can be removed from the expression without changing the result.

[5 marks]

Question 2

2.1 Prove the following identities.

a: $\sin^2\alpha + \cos^2\alpha = 1$

b: $1 + \tan^2(\alpha) = \dfrac{1}{\sin^2(\alpha)}$

[4 marks]

2.2 Find the exact values of the trigonometric functions $\cos(\alpha), \sin(\alpha)$ and $\tan(\alpha)$ the following angles (do not use your calculator to find the values)

a. $\alpha = 45^o$

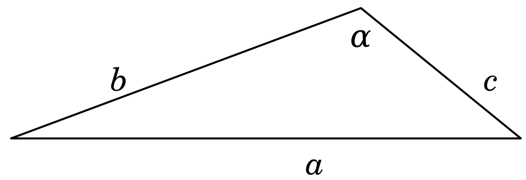
b. $\alpha = 60^o$

c. $\alpha = -30^o$

[6 marks]

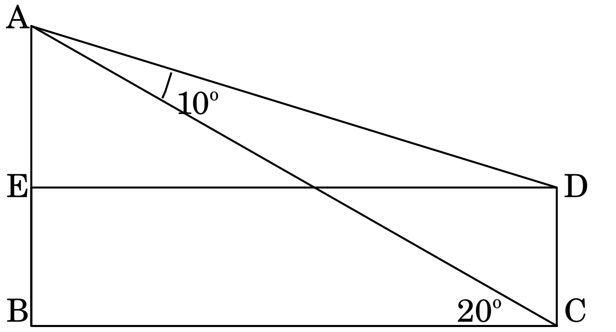
2.3 Use the following diagram to prove that $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$.

[7 marks]



2.4 In the following diagram, find the distance AB, given that:
CB=DE; EB=DC=25; Angle EDA=10°; angle BCA= 20°

[8 marks]



Question 3

3.1 Given the vectors **a**, **b** and **c** compute the following vector equations.

$$\mathbf{a}=\begin{bmatrix}5\\4\\2\end{bmatrix}; \mathbf{b}=\begin{bmatrix}0\\1\\-2\end{bmatrix}; \mathbf{c}=\begin{bmatrix}1\\2\\1\end{bmatrix}$$

a: $\mathbf{a}-\mathbf{b}+\mathbf{c}$

b: $\mathbf{c} \times \mathbf{c}$

c: $\mathbf{a} \cdot \mathbf{c}$ **[4 marks]**

3.2 If $\mathbf{a}=3\mathbf{j}+4\mathbf{k}$ and $\mathbf{b}=2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$ calculate the following.

a: $\|\mathbf{a}\|$

b: $\|\mathbf{b}\|$

c: $\|\mathbf{a}\|\mathbf{b}$ **[6 marks]**

3.3 Given $\mathbf{r}=a_r\mathbf{i}+b_r\mathbf{j}+c_r\mathbf{k}$ and $\mathbf{s}=a_s\mathbf{i}+b_s\mathbf{j}+c_s\mathbf{k}$ and $\mathbf{r}\ \mathbf{s}=\|\mathbf{r}\|\|\mathbf{s}\|\cos\alpha$, where α is the angle between $\mathbf{r}$ and $\mathbf{s}$, show that $\mathbf{r}\,.\mathbf{s}=a_r a_s+b_r b_s+c_r c_s$.

[5 marks]

3.4 Using the scalar product calculate the angle between **r** and **s** where $\mathbf{r}=2\mathbf{i}+3\mathbf{j}+6\mathbf{k}$ and $\mathbf{s}=\mathbf{i}-4\mathbf{j}\text{-}2\mathbf{k}$. **[5 marks]**

3.5 Describe how the scalar product is useful in identifying back-facing polygons.

[5 marks]

Question 4

4.1 Give the algebraic equivalent of the following matrices.

a: $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

b: $$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -2 & 0 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

c: $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[4 marks]

4.2 Describe the following 2D matrices and give an example of each.
a: a scaling matrix
b: a rotation matrix
c: a shear matrix
d: a translation matrix. **[8 marks]**

4.3 Describe how homogeneous coordinates provide a mechanism for combining scaling and translation matrices. Illustrate your answer with an example.

[5 marks]

4.4 Derive a single matrix to undertake the following collective 2D transformations. A scaling by a factor of 2 about the point P(1,-1), followed by a translation by 5 in the x-direction and 3 in the y-direction.

[8 marks]

Question 5

5.1 Compute the value of X_b for the following where the subscript represents the base of the number.

a: $X_3 = 32_9$
b: $X_4 = 3_{10}\sqrt{100_5}$
c: $X_{16} = 27_{10}$
d: $X_5 = 121_5 + 101_3$
e: $X_7 = 14_{10}$ **[5 marks]**

5.2 Explain briefly how to normalize a vector, and why it is useful to have unit vectors in scene calculations.

[5 marks]

5.3 Define the meaning of the following and give an example of each.

a: an irrational number
b: a vector
c: a complex number
d: 3D vertex

[5 marks]

5.4 Illustrate how $\sqrt{-1}$ rotates a complex number through 90º.

[5 marks]

5.5 Simplify the following complex numbers.
a: $(1+3\mathrm{i})+(2-4\mathrm{i})$
b: $(2-3\mathrm{i})-(1-2\mathrm{i})$
c: $(2+i)(1-i)$
d: $(2+i)^2$
e: $i^2+i^4+i^6+i^8$

[5 marks]

Question 6

6.1 Describe the geometric meaning of a,b and c in the Cartesian form of the line equation $ax + b = c$ and illustrate your answer with an example.

[5 marks]

6.2 Find the equation of the line passing through the points P(-1 , -1) and Q(2 , 1); write the answer in Hessian normal form

[5 marks]

6.3 Compute the shortest distance from the origin to the plane defined by equation $3x + 2y - 4z = 10$.

[5 marks]

6.4 Describe how line equations can be used to determine whether points are inside, outside, or located at an edge or vertex of a convex

[5 marks]

6.5 Find an equation of the line that passes through the point (-2 , 1) and is parallel to the line $x - y = 2$. Write the answer in Hessian normal form

[5 marks]

20/02/2007
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