Masters Thesis

Procedural Fish Animation

Adam R.C. Gritt
MSc Computer Animation & Visual Effects

September 11, 2010

Acknowledgements

Jon Macey for running a fantastic course

Sola Aina for invaluable help with the Maya API

All NCCA staff for insightful and stimulating lectures throughout the year

Contents

List of Figures

Chapter 1

Introduction

Computer animation remains one of the most labour-intensive areas in the visual effects industry (Ltd., 2005, p2). Our ability to perceive artificial movement is extremely sensitive. Furthermore, even if visuals are obviously artificial (i.e. not attempting to be photorealistic) they do not typically break the fourth wall. Contrastingly, incorrect animation immediately reveals itself to the view, especially in the case of humanoid characters. 'Ragdoll' physics has long been used to simulate human animation in video games, but it is vastly inferior to manual keyframe animation. NaturalMotion's Euphoria engine, "based on a full simulation of the 3D character, including body, muscles and motor nervous system" (NaturalMotion Ltd., 2009), is a more modern approach and achieves far more realistic results. Perhaps our visual perception's sensitivity is founded on an attunement to musculoskeletal constraints.

Indeed, the modern approach to animation uses a combination of musculoskeletal constraints and manual keyframing in order to achieve the best results. A typical example would use a skeletal rig as a basis for animation, with muscles being a secondary and automatic approach to skin deformation. In this sense, the muscles are not really muscles at all; they do not exert force. In order to use the muscles in this way, you would have to fully understand the biomechanics of a real creature. This task would be extremely expensive, and is perhaps the reason that a system such as Euphoria only came about as recently as 2006. Why make this investment when you could generate hundreds of hours of animation manually for the same cost? Euphoria is principally used in video games, interactive environments where the animation must be generated on-the-fly. Similarly a flock or birds or shoal of fish contains hundreds or thousands of entities and generating this animation manually would be cost-prohibitive.

Procedural animation overcomes all scalability limitations, but with the cost of a large development overhead (Ltd., 2005, p3). It also tends to lack the uniqueness and personality of manual animation. Its use on the big screen is usually restricted to background elements, with the lead characters still being hand-animated to emphasize their idiosyncracies.

Chapter 2

Previous Work

The first widespread use in realtime applications was the aforementioned ragdoll physics. Its level of realism was quite poor, but it was fast enough to run in realtime on hardware of the early 2000's. Both ragdoll and Euphoria are designed with humanoid figures in mind; although their principles can be applied to other creatures such as quadrupeds, it would require significant additional work.

Figure 2.1: Simulation of a human falling down stairs using ragdoll physics *(Source: Animats, 1997)*.

Pixar's *Finding Nemo* made extensive use of procedural animation for its jellyfish and coral animation (Cohen, 2003, p4). Whilst animation of jellyfish 'bells' were "sometimes hand-animated" the vast majority of the 77,000 in the film were handled automatically. The hanging tentacles from each jellyfish and sea anemone tentacles, both heavily affected by the water current, were also animated procedurally. Manual animation would have been particularly difficult because the many tentacles must never be static or intersect with each other. In both cases the automatically generated animation was used only for background elements. Shoaling and schooling, analagous to bird flocking, were also handled procedurally (Teo, 2003, p6). Whilst this is not

creature animation in itself, each fish in the shoal must be animated individually so that it appears to be swimming in the appropriate direction. Pixar took this a step further by using schools of fish to form the shape of various objects *(see figure 2.2)*.

Figure 2.2: The shape of a ship is formed by a shoal of fish in *Finding Nemo* (Pixar Animation Studios, 2003).

Wu & Popović (2003) developed a method for synthesizing realistic bird flight animation. Their work was able to produce a number of aerial maneuvers "including taking off, cruising, rapidly descending, turning and landing" (p1) with good accuracy. Using 2003 hardware they were only able to achieve this at 1/3000th of realtime (Wu & Popović, 2003, p6), which would limit the tool's use to an artist.

Tu & Terzopoulos (1994) released a paper describing methods to mimic the locomotion and behaviour of fish artificially. They pay particular attention to how fish react to external factors E; $R \subseteq E$ where R is the set of factors a fish is aware of through its sensory perception (p4). For animation purposes, these external factors are obstacles or prey to avoid and fish to shoal with. Tu & Terzopoulos define an area around the fish, similar to the neighbourhoods described by Reynolds (1987, p6). For simplicity, Tu & Terzopoulos use an area around the fish of fixed radius spanning 300 degrees, the excluded area represents an area the fish could not see due to the position of its eyes.

Chapter 3

Technical Background

The goal for this project is to develop a procedural animation framework for fish movement. Such a framework would remove any scale-based restrictions on animation, as the process is entirely automated.

3.1 Maya

The framework will be built using Maya. Whilst this presents many unique challenges, there are also significant advantages to developing with Maya. Tools developed using one of the Maya APIs benefit from automatic save state handling integration with other Maya tools and renderers. Maya has a complete and familiar interface and using the API eliminates the need to develop one as would be necessary in a standalone application.

3.1.1 Scripting

There are two distinct development methods for Maya; these are interpreted scripting, and the C++ API. The scripting interface supports both MEL (Maya Embedded Language) and to a lesser extent, Python. As with most scripting implementations, Maya uses an intepreter meaning that there is a significant performance loss compared to native code. This is offset by its ease of use; code doesn't need to be compiled or loaded beforehand, as must be done with C++. Use of scripting is also inappropriate for commercial software, since the source code is required for it to run. This would allow anyone who purchases a product to see and understand the secrets behind the software.

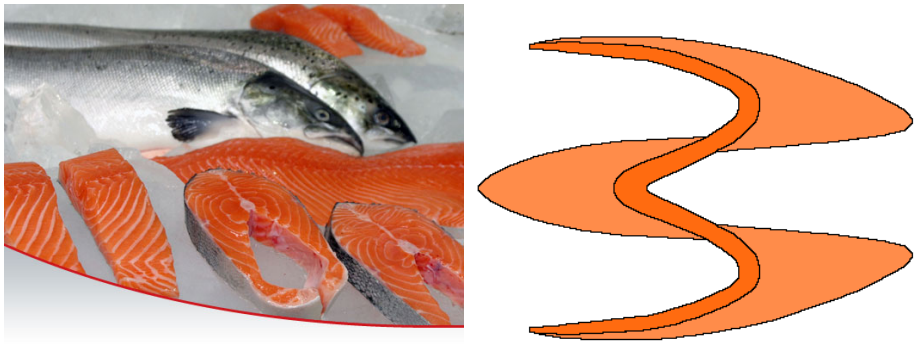
3.1.2 C++ API

The alternative to scripting is the Maya C++ API, which offers maximum performance through use of native code but has a very rescrictive development environement. Specific compilers must be used to build plugins for differing versions and operating systems. For example, Maya 2011 for Windows x64 requires that plugins be compiled with VC9, Visual Studio 2008's compiler. Additional effort is therefore required to ensure cross-platform compatibility, contrasting with the scripting API which has no such disadvantage. Although the plugin architecture allows source

code to be kept hidden, this combined with the compiler restrictions means that plugins are not future proof. A new version of Maya is likely to require the source code for a new compilation of the software.

3.2 Fish Biomechanics

As with existing procedural animation technologies, the biomechanics of the subject must be understood in order to generate realistic results. The most obvious method would be to model every muscle in a fish, with the effects of contraction and relaxation propogating around the body.



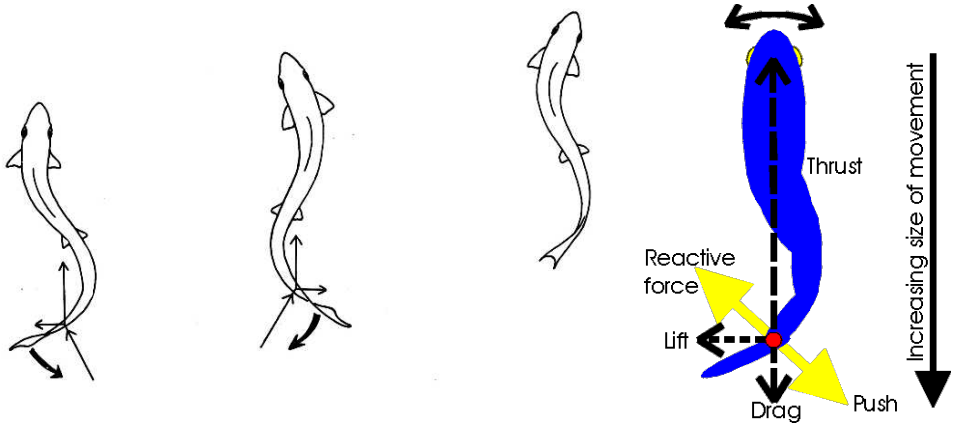
(a) Salmon

(b) A single salmon myotome *(Source: Ramel, 2009)*

Figure 3.1: Salmon muscles

Figure 3.1 shows the pattern of muscles in fish, in this case salmon. The white lines in salmon meat *(3.1a)* show layers of fat separating individual myotomes *(3.1b)*, a type of muscle. There are dozens of myotomes in a single fish, and it's clear that these are not the simple, straight, elastic muscles found in other animals. The unusual shape and curvature would make modelling the propogation of force extremely difficult. Even if such a model were to be produced, there would probably be significant performance limitations due to its complexity.

Rather than engineering an accurate model of a fish, its movement must be examined in order to produce a reverse-engineered solution. Figure 3.2a shows a fish with at least two independent sets of muscles, allowing it produce an 'S' shape which increases the surface area which can be used to push water *(figure 3.2b))*. Other fish can range from having hardly any visible muscle movement (e.g. goldfish) to having even more pronounced curvature (eels).



(a) Movement cycle of a fish *(Source: Mackean, 2010)*.

(b) Force of a fish swimming *(Source: Buchheim, 2006)*.

Figure 3.2: A fish using its tail as a paddle to generate forward momentum.

Chapter 4

Methods

Having ruled out an accurate biomechanical model due to the complexity of myotome modelling, an alternative approach is to use a geometric technique. This is the technique used by Tu & Terzopoulos and uses a visually accurate model as a basis for deformations. The model is deformed using a spring-mass system, with specific connections defined as affectors, i.e. muscles.

4.1 Harmonic Oscillators

Newtonian mechanics defines a harmonic oscillator as a system which exhibits a restoring force F which is proportional to the displacement from its equilibrium x. Hookes law states:

$$F = -kx$$

Where k is a positive constant. The spring's equilibrium position is given as $x = 0$. This can otherwise be expressed as:

$$F = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx$$

The system generates a force proportionate to the amount which it is stretched. Such a system, when displaced from its equilibrium or rest position, undergoes continual sinusoidal oscillations about this point; the system never repeats infinitely and never comes to a rest *(figure 4.1, $\zeta = 0$)*.

4.1.1 Spring

A spring can be defined as a harmonic oscillator connecting two points. Each point has a mass, although in some cases a point is treated as being immovable (infinite mass). The mass of the spring itself is treated as being negligible. The constant k becomes an effective 'stiffness' measurement, with higher values resulting in a greater restorative force, increasing the frequency of oscillations.

4.1.2 Damping

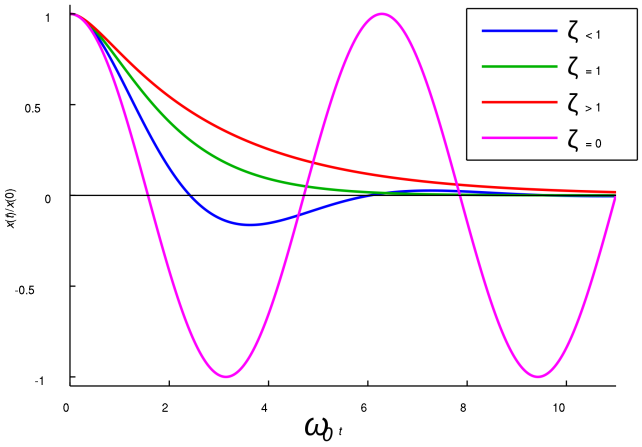


Figure 4.1: Effect of damping on a harmonic oscillator *(Source: Nogueira (2007))*

In reality, a spring would lose its energy through air displacement and internal friction. To model this mathematically, a damping ratio is applied to the force at regular intervals, eventually reducing the energy in the system to zero, at which point it will have found a new equilibrium *(figure 4.1,* $\zeta < 1$*)*. Other cases may require the system to return to equilibrium without oscillating *(figure 4.1,* $\zeta >= 1$*)*.

4.1.3 Muscle

In biomechanics, skeletal muscle is analagous to an overdamped harmonic oscillator *(figure 4.1,* $\zeta > 1$*)*. Upon contraction the rest length or spring constant, k, is altered, and the muscle quickly finds its new position. In the virtual model, the spring can be defined as a function of the two nodes which it connects. When a spring exerts force the two connected nodes are displaced proportionately to their mass, contrasting with typical systems which may have an anchor such as a bone.

Chapter 5

Solution

5.1 Spring-Mass System

A spring-mass system can be defined by set of nodes N and springs S. Let each node have a mass m and position p. Let each spring have a stiffness c, and have it's rest length l defined as the vector between two nodes $l = p_i - p_j$. Such a system is inherently stable until some external force is applied, since all springs remain at their rest lengths. The force applied to each node by the springs connected to it is:

$$F = \sum_{\text{s} \in \text{springs}} c_s(p_i - p_j - l_s)$$

A damping factor should be applied proportionately to the force exerted by springs, and also relative to the change in time between simulation updates.

5.2 Data Structure

The data structure of the nodes and springs proved to be difficult. Due to the procedural nature of skeletal construction *(see 5.3)*, it was not possible to assign all of a node's springs at instantiation. Each node contains a vector of type NodeConnection, which contains a pointer to a spring, and the other node on that spring. This structure allows maintenance of nodes to be encapsulated by their parent SkeletonSection objects.

5.3 Skeleton Structure

The skeleton of the virtual fish is divided into segments which have predefined connectivity rules and appearance. The system is designed to be fully extensible and should allow additional segment types to be added provided that the appropriate connectively rules are defined. Figure 5.1 illustrates the composition of each of the predefined segments.

Segment	Nodes	Total Springs	Structural Springs	Multiplicity
Nose	1	0	0	1
Body	4	6	2	$1 \ldots n$
Tail	2	1	1	1

Figure 5.1: Skeleton components of the virtual fish.

A skeleton is constructed by adding segments to it in turn. Construction of the spring network is handled automatically, as defined by the rules of each specific segment type.

Connection	Total Springs	Structural Springs	Muscle Springs
Nose-Body	4	0	0
Nose-Muscle	4	0	0
Body-Body	12	8	0
Body-Muscle	12	8	0
Muscle-Muscle	12	8	4
Body-Tail	8	4	0
Muscle-Tail	8	4	0

Figure 5.2: Connectivity between components.

5.4 Maya Integration

The greatest challenge in development of the software was integration with Maya. The Maya C++ API can be used to add functionality to Maya in two ways; through definition of new commands which perform a certain function within the scene on existing nodes, or through definition of a new node type. Code must follow a very specific pattern in order to function properly.

5.4.1 Nodes and Attributes

Maya represents scenes through the dependency graph - a network of nodes, each containing numerous attributes. This network is responsible for Maya's ability to save to an ASCII-based file format. The undo/redo functions are also heavily dependent on a network representation in order to work properly. Most importantly though, modifying a node anywhere in the scene hierarchy results in a representative change in all child nodes; if the extrude node were to be deleted in a PolySphere-¿Transform-¿Extrude hierarchy, the extrude operation would still function as intended.

Each node also contains a number of attributes, these are all handled internally by Maya so that they inherit functionality such as keyframing and expressions. When attributes are changed, the dependency graph is used to check whether any other attributes are affected as a result. For example when a Length attribute is changed, the Volume attribute would need to be updated; however instead of calculating the new value, Maya marks the Volume attribute as 'dirty'. These dirty attributes are

then only recalculated when read, and this results in a very substantial performance gain. For this reason it is necessary to manually describe the relationships between attributes when using the C++ API. This is a large development overhead for a task which would normally be taken care of by an optimising compiler.

Maya has a non-standard way of defining node attributes. Whilst in a typical C++ program one might define use the following to define an array of doubles:

```
1 double m_springConstants [96];
```

Maya requires its own declaration for the template definition which allows it to make new node instances. The following shows the equivalent code in the Maya API:

```
1 MObject m_springConstants;
2
3 MStatus Node::initialize()
4 {
5   MFnNumericAttribute numAttr;
6   m_springConstants = numAttr.create("springConstants", "spring",
        MFNNumericAttribute::kDouble, 0.0);
7   numAttr.setArray(true);
8   addAttribute(m_springConstants);
9 }
```

This leads to highly verbose and somewhat cluttered code. Array attributes both defined and processed differently than standard attributes. Further still, compound attributes are used to define data structures which contain more than one type of variable. This is in stark contrast with the typical approach when using an object oriented language such as C++, which is to use objects or type definitions. In addition to requiring a custom attribute definition procedure, Maya also presents a specific API for reading and writing attributes. Each node has a datablock, an object which contains the data for all of the node's attributes. This datablock must be navigated using the API in order to retreive and modify data.

Aside from the attributes necessary to perform updates to the simulation, two global constants are also defined in the plugin. These are the 'speed' and 'force' of the animation. A higher speed results in a higher frequency of oscillation between left and right side muscle contractions. Similarly, a higher force results in a more significant reduction in the muscle spring rest length, resulting in a more pronounced swimming movement.

One of the most challenging aspects of development is full integration with Maya's animation controls. Maya allows the user to scrub forwards and backwards in time, and skip to any point. This contrasts with a typical OpenGL application which plays continuously in real-time. For algorithms which aren't reversible, this presents a problem; the original state must be stored in order for the scene to be reproducible. Additionally, Maya does not supply the typical Δt variable typically seen in OpenGL applications, instead only the current time is supplied.

5.4.2 Node Implementation

In order to integrate with Maya's timeline, a node must connect with a time node. Every new empty Maya scene contains a time node, 'time1', however additional time nodes can be made if desired. The outTime attribute of a time node supplies data

in the format MTime. An attribute of this type must then be added to the custom node in order to receive this data, an inTime attribute. Node definitions are merely a static template which Maya uses in order to create new instances of a node. Since the definition of a node is static, it cannot perform any operations on new instances of the node. A dependency graph connection is required between the time node's outTime attribute and the new fish node's inTime, however this cannot be done from within the fish node definition.

5.4.3 Dependency Graph Modification

There are two methods of performing such a connection from outside the node, either the connectAttr MEL command can be used, or a custom API command can be called from MEL. As with all MEL, the connectAttr command is itself just an interface to an equivalent call in Maya's C API. For non-trivial dependency graph operations, such as a series of connections being made procedurally, it is logical to hide this logic within a custom API command.

5.4.4 Custom API Command

The MPxCommand API class is used as a basis for new MEL commands. A combination of nodes and commands can be integrated into the same binary plugin, a technique which allows attributes to be linked through reference rather than by their name. Commands can also be called with an array of arguments for specifying options such as the number of fish to be created.

For each attribute that is connected an MPlug object must be retrieved. This is a reference to the instance of an attribute, and provides an interface for accessing data and being connected in the dependency graph. The MDGModifier class is used to perform operations on a scene's dependency graph. This is then used to procedurally connect each attribute, for example connecting the outTime attribute of the time1 node to the inTime attributes of a dozen fish nodes. Attributes can supply data to any number of other attributes in a one-to-many relationship, but only receive their data from a single attribute. The connection between the attributes can be thought of as a pointer is in the C language; the inTime attribute literally refers to another attribute and the data each contains is always identical.

5.4.5 Skeletal Mesh

The topology of the skeletal mesh does not conform to any typical mesh structures included in Maya. The skeleton is essentially just a collection of lines, with neither vertices nor polygons. Springs can overlap in an X-shaped arrangement, a topology which is incompatible with normal meshes. Hence the visualisation of the skeleton requires custom code to be written. The MPxLocatorNode class intended as the basis for nodes of this type; shapes that are drawn on the screen but not rendered.

Maya uses OpenGL for its viewport, and any OpenGL calls from within a Maya plugin are invoked within this viewport. The MPxLocatorNode class templates a draw function which is automatically called by Maya when necessary. A scene redraw occurs whenever something changes in the scene, in particular the location of the

camera. This function is also used as the basis for selection within the GUI, OpenGL's selection mechanism is used to determine whether the object gets selected by a particular mouse event.

5.5 Spring-Mass Calculation

With the stable fish structure being defined, the system finds equilibrium at its original position. Muscle contractions are defined by the muscle springs in the procedurally generated spring-mass system. Muscles form two groups, one for each side of the fish, and these are actuated as a group by the system.

The final solution for the system is:

$$F_i = m_i \frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} + \zeta \frac{\mathrm{d}x_i}{\mathrm{d}t} - s_i;\ i = 0, \ldots, n$$

Where x is the node position, i is the set of nodes, n is the number of nodes in the system and where:

$$s_i = \sum_{\mathrm{j} \in \mathrm{N_i}} F_i^s j(t)$$

Where s denotes the force applied to the node as a result of connected springs; the sum of the force springs connecting node i to its set of connected nodes N. The damping factor, ζ is applied proportionately to time.

Whilst Newton's Second Law allows us to calculate the force applied on a mass at any given point in time, it does not account for constantly changing variables; variables that are a function of time. The most sigificant effect of this is easily illustrated with a spring analogy. As a spring returns from its maximum extent, it has both a high velocity and high acceleration. If the Δt of the solver is too high, the node may completely pass it's maximum negative extent before the scene is next updated and drawn. As this continues, the spring is calculated as having progressively more energy. In a system of springs, this would rapidly cause the structure to break down, as the energy of the problem spring easily outweighs any possible restoring force by the rest of the system.

In animation, the ideal Δt is easily calculated as the time between frames. This results in the lowest number of updates being performed, but in some cases it is not always possible. The default state of the procedurally generated mesh was tested with a Δt of $1s/FPS = 0.04s$. Whilst this results in a stable mesh, the results are neither consistent nor symmetrical. In order to alleviate this problem, the default Δt is broken down into several smaller time-steps. The system now performs 10 updates, each with $\Delta t = (1s/FPS)/10 = 0.004s$. The obvious disadvantage to this is that the system will be nearly 10 times slower, however this is also the most efficient solution *(see chapter 6)*. For the same reason, the solver is not capable of directly jumping between frames. This, again, is easily resolved by performing the equivalent amount of updates with a smaller time-step.

Problems also occur when extreme values are used for the speed and force attributes. When the spring-mass system reaches its maximum extent, i.e. when the

muscles which have been contracted are closest to their preferred rest position, other springs which should maintain their structure begin to change. This is because the system can no longer find resitution through movement in the muscle springs and therefore does so in the weakest structural springs. This problem can be avoided by not specifying extreme values for these attributes. This is not always desirable, however. In Pixar's *Finding Nemo*, for example, many of the characters are animated with extreme movement to convey their personality.

5.6 Results

Figure 5.3 illustrates the system upon initialisation. The custom spring-mass mesh is generated procedurally and remains in its equilibrium state. After a period of time has elapsed, specified by the speed attribute, the fish will contract its muscles on the right side. Upon the next update of the system *(not the same as the next frame, see section 5.5)*, the fish will being to bend as if those muscles were contracted. The change in position is most dramatic initially; since the rest length of the muscles have been changed but the actual length has not. Since the spring's actual length is farthest from it's rest length, the associated mass (i.e. nodes) experience the most dramatic restoring force at this point.

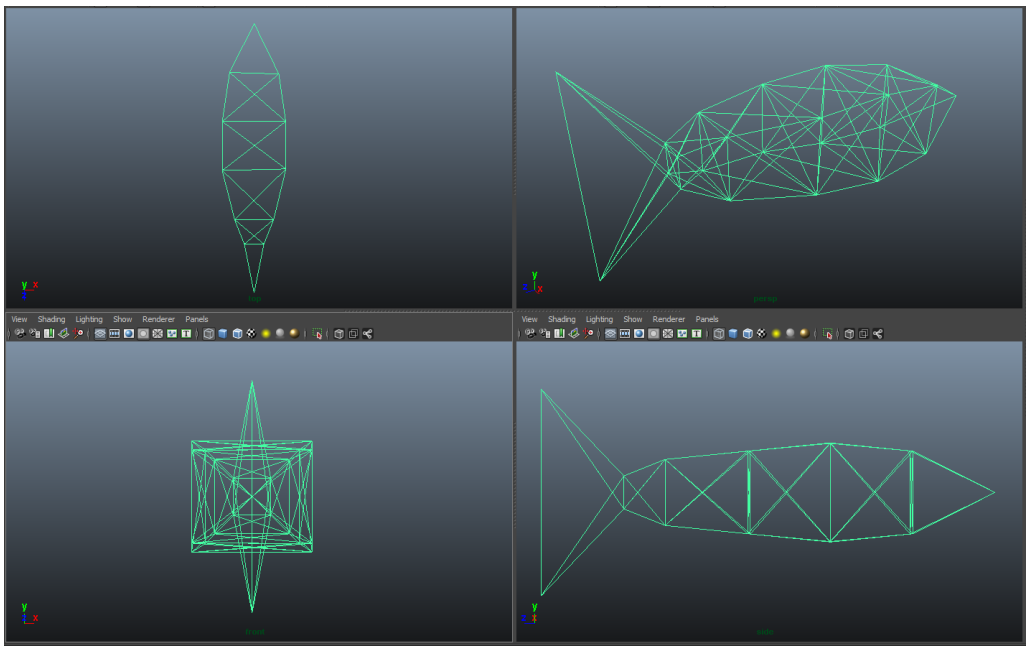


Figure 5.3: The procedurally generated spring-mass mesh in its equilibrium state.

Figure 5.4 shows the system in an animated state; the right side muscles have been contracted and those springs approach their rest position. As the springs approach their rest position, they experience the least restorative force, indeed there is hardly any movement in the system. The values for force and speed should be specified

such that this period of non-movement is as desired. Most fish will have a very short period where neither side of their myotomes has been contracted.

Upon a second elapsation of the time specified by the speed attribute, the opposite (left-side) muscles will contract, and the right side muscles will regain their initial values for rest length. The effect of this is two fold; the muscles on the right side of the body will extend along the axis of the fish, and the muscles on the left side will shrink, resulting in the characteristic curvature seen in swimming fish.

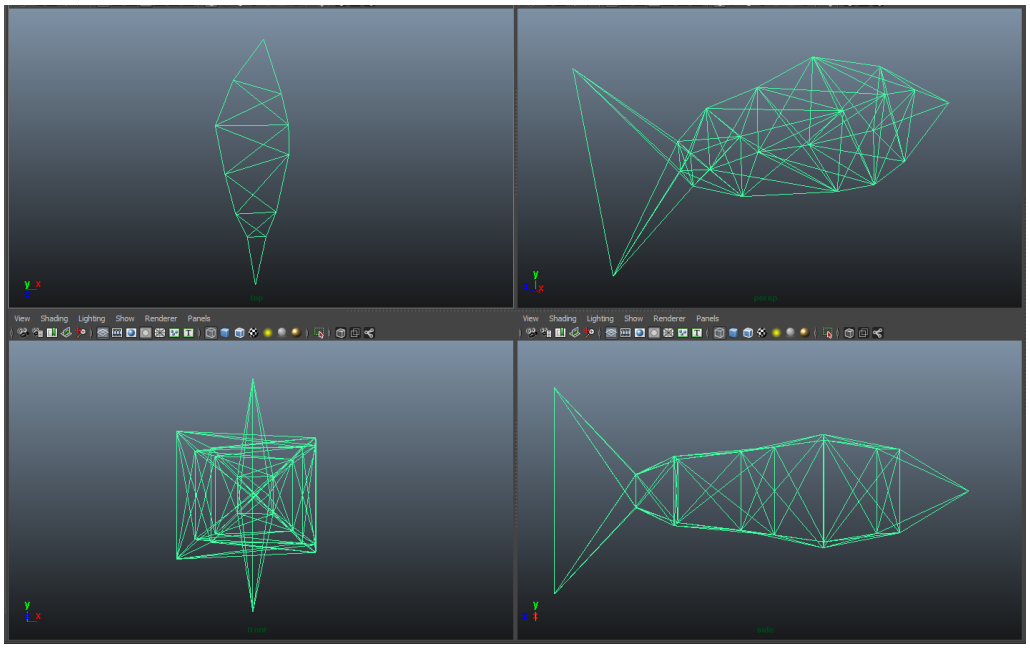


Figure 5.4: The system in an animated state. Muscles on the right side have been contracted, whilst muscles on the left are relaxed.

Chapter 6

Conclusion

The procedural animation works well, with the system creating animation entirely procedurally. The animation is quite realistic, however the fish is only able to swim sinusoidally forward. With some additional work the system would be able to generate animation which appears to be swimming in a particular direction.

The project was very challenging, specifically because the Maya API has a steep learning curve. Maya has a very unique structure, and the documentation can be hard to follow. Maya presents a very limited model of it's internal structure through its API. This is a result of its dependence on backwards compatibility. With less of the internal structure exposed, the Maya developers have greater freedom to perform internal changes without affecting the way it behaves. Instances of the class MObject can refer to attributes, nodes, transforms, vertices, etc; one might expect a class called MAttribute or MNode to inherit from MObject, but this is not the case. Care must be taken during development to keep track of the true type of an MObject so that it can be attached to the correct function set.

Even with a full understanding of the API, coding for a Maya plugin is extremely verbose and long-winded compared to other libraries or frameworks. That said, once completed, Maya plugins can integrate extremely well and take advantage of the vast quantity of existing scripts, plugins and built-in functionality that Maya has to offer. Much of Maya's functionality is developed in the same way as third party plugins.

The implementation that was developed solves the spring-mass system using Newton's Second Law. This results in the aforementioned boundary problems assosciated with extreme input, and can prevent certain resolutions of the system from being reached. Tu & Terzopoulos solve this problem using "a numerically stable, implicit Euler method". Whilst this is an ideal solution, such a solution requires the use of a system of ordinary differential equations (ODEs) in order to concurrently find the equilibrium position for all springs in the system. This is a somewhat different approach to the muscle actuation system in the procedural spring-mass system; the approach of Tu & Terzopoulos finds a position of absolute restitution for the system. This means that rather than changing the rest length of the muscle springs once per real-life muscle contraction, the rest length itself must be calculated as a function of time. Rather than allowing the system to gradually come to a new state of equilibrium (the speed at which this occurs is limited by the damping factor), the system is solved completely. A continually changing rest length produces the effect

of animation even though the system is not in a state of flux. A system of ODEs is also far slower than basing an implementation on Newton's Second Law, even when that system must be solved multiple times for each time step.

Overall the project was quite successful, however it is important to integrate the functionality into a more complex system to make full use of it. A flocking or shoaling system would make best use of the procedural nature of the system, facilitating the animation of an unlimited number of fish with minimal effort. The high computational performance of the developed system means that it would be perfect for such use. Another major strength of the system is the modular design of the fish construction. With little effort, the system could be adapted to produce animation for other fish-like creatures such as sharks, whales and eels.

Bibliography

References

Animats. 1997. *Ragdoll falling downstairs*. http://www.animats.com/.

Buchheim, Jason. 2006 (June). *A Quick Course in Ichthyology*. http://www.marinebiology.org/fish.htm.

Cohen, Karl. 2003. *Finding the Right CG Water and Fish in Nemo*. `http://www.awn.com/articles/technology/finding-right-cg-water-and-fish-inemoi`.

Ltd., NaturalMotion. 2005. *Dynamic Motion Synthesis*. `http://www.naturalmotion.com/files/white_paper_dms.pdf`.

Mackean, D G. 2010 (August). *Fish Swimming*. http://www.biology-resources.com/drawing-fish-swimming.html.

NaturalMotion Ltd. 2009. *NaturalMotion euphoria*. `http://www.naturalmotion.com/euphoria.htm`. 24th Sept.

Nogueira, Nuno. 2007 (September). *Damping*. http://en.wikipedia.org/wiki/File:Damping.svg.

Pixar Animation Studios. 2003 (May). *Finding Nemo*.

Ramel, Gordon. 2009 (December). *Fish Muscles*. http://www.earthlife.net/fish/muscles.html.

Reynolds, Craig W. 1987. Flocks, herds and schools: A distributed behavioral model. *Pages 25–34 of: SIGGRAPH '87: Proceedings of the 14th annual conference on Computer graphics and interactive techniques*. New York, NY, USA: ACM.

Teo, Leonard. 2003. *The Making of Finding Nemo*. `http://features.cgsociety.org/story_custom.php?story_id=1389`.

Tu, Xiaoyuan, & Terzopoulos, Demetri. 1994. Artificial fishes: physics, locomotion, perception, behavior. *Pages 43–50 of: SIGGRAPH '94: Proceedings of the 21st annual conference on Computer graphics and interactive techniques*. New York, NY, USA: ACM.

Wu, Jia-chi, & Popović, Zoran. 2003. Realistic modeling of bird flight animations. *Pages 888–895 of: SIGGRAPH '03: ACM SIGGRAPH 2003 Papers*. New York, NY, USA: ACM.