Master Project

Development of a car physics engine for games

(Photo courtesy of Need for Speed Most Wanted – BMW M3)

Abstract

Car racing games are a lot of fun. You can drive high-powered vehicles that you could not possibly afford in real life. Between the speed, the power, and the fiery crashes, you can create many exciting game scenarios using cars. Whether you are developing a car racing game based on realistic physics or a car racing game based on liberal physics, it is important to have a robust car physics engine for your game.

This project is useful as it could be widely used in various fields such as games, animations, and simulations depending on the configurations of the physics engine. Last but not least, it helps you understand the physics that underlies the dynamics of cars.

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Chapter 1: Introduction

This thesis documents the design and implementation of an efficient yet pleasant car physics engine for games, which includes acceleration, braking, cornering, weight transfer, suspension system, and collision responses. With the physics engine, we can simulate various interesting visual effects such as cornering at high speed and drifting.

In chapter 2, some examples of available car physics engine are mentioned.

Chapter 3 explains the forces acting on a car and how to calculate them. In chapter 4, engine torque, wheel torque, gear, and the transmission system are introduced. In chapter 5, the physics of cornering as well as the approximation of the lateral forces on each wheel are illustrated. Chapter 6 documents the conceptual design for the first part of the car physics engine. Chapter 7 covered the concept of weight transfer and suspension system, which helps add sophisticated effects to the car. Chapter 8 demonstrates the implementation of the second part of the car physics engine using the knowledge from chapter 7. Chapter 9 shows the implementation of projectiles and collision responses from wall. Chapter 10 covered the implementation of collision response from two moving cars. Chapter 11 documents the structure of the car physics engine. Chapter 12 presents results and gives an idea over potential applications. The results are evaluated in chapter 13 where limitations and future work are briefly discussed.

Chapter 2: Related Work

Car physics engine is a big topic especially in the field of games. There are many car physics simulation software available. However, most of the high quality racing simulators is not for free or free for non-commercial use only.

Racer (2000) is a free car simulation engine (for non-commercial use), using real car physics and motion formulae from actual engineering documents to get a realistic feeling. It uses OpenGL for rendering and produces commercial-quality moving image. The engine supports 2-8 wheels vehicles with 6 degrees of freedom for the chassis movement and 15 degrees of freedom in a total car (wheels, engine, clutch etc).

CarX (2008) is a commercial high quality car physics engine using real cars to reconcile the behavior of vehicles in the game with real life. The engine supports different types of suspension (McPherson, double wishbone, and dependent) and different types of car from small cars to trucks with a large number of wheels. It supports different gear types, which are FWD, RWD, and 4WD. Another interesting feature of CarX is that it simulates visual deformation of tires. It also supports different surfaces (asphalt, snow, ice, sand, grass, and earth).

GTR and Need for Speed are examples of successful car racing games. In GTR player experiences authentic racing experiences where cars and circuits behave, as they would do in real life. Need for speed, on the other hand, is an arcade game rather than a simulation game. It, therefore, relies more on liberal physics rather than realistic physics.

Chapter 3: Forces

To simulate cars in game, we need to compute the forces acting on a car. Longitudinal, lateral, and vertical forces should be handled separately. Longitudinal forces operate in the direction of the car body (or in exact opposite direction). These are wheel force, braking force, rolling resistance, and drag (air resistance). Together these forces control the acceleration or deceleration of the car. Lateral forces, which are caused by sideways friction on the wheels, allow the car to turn. The force of gravity (vertical), which acts both normal to the inclined surface, as well as parallel to the slope, also influences the car.

3.1 Aerodynamic drag

When the car is in motion, an aerodynamic drag will develop that will resist the motion of the car. Drag force is proportional to the square of the velocity and the formula to calculate the force is as follows.

$$F_{drag} = -C_{drag} * V * |V| \tag{3-1}$$

Where V is the velocity vector and C_{drag} is a constant, which is proportional to the frontal area of the car.

3.2 Rolling resistance

Rolling resistance is caused by friction between the rubber and road surfaces as the wheels roll along and is proportional to the velocity.

$$F_{rr} = -C_{rr} * V \tag{3-2}$$

Where V is the velocity vector and C_{rr} is a constant

3.3 Gravity

Gravity pulls the car towards the earth. The parallel component of gravitational force, $F_g = mg\sin\theta$ (θ is the angle of the slope), can pull the car either forwards or backwards depending on whether the car is pointing uphill or downhill.

3.4 Traction

The engine generates torque, which when applied to the wheels causes them to rotate. Friction between the tires and the ground resists this motion, resulting in a force applied to the tires in the direction opposite to the rotation of the tires.

$$F_{traction} = T_{wheel} / R_{wheel} \tag{3-3}$$

Where $F_{traction}$ is the force applied to the tires, T_{wheel} is the torque applied to the wheels, and R_{wheel} is the wheel radius. I will discuss how to calculate the torque applied to the wheels in the “Engine torque” section. The lateral forces will be discussed in the “Cornering” section.

3.5 Updating the position of the car

To determine the position of the car first we must find the net force on the car. The total longitudinal force is the vector sum of these four forces.

$$F_{long} = F_{traction} + F_{drag} + F_{rr} + F_g \qquad (3\text{-}4)$$

When braking, a braking force replaces the traction force, which is oriented in the opposite direction. A simple model of braking is as follows.

$$F_{braking} = - u * C_{braking} \qquad (3\text{-}5)$$

Where u is a unit vector in the direction of the car's heading and $C_{braking}$ is a constant.

The acceleration of the car is determined by the net force on the car and the car's mass via Newton's second law.

$$a = F / M \qquad (3\text{-}6)$$

The car's velocity is determined by integrating the acceleration over time using the numerical method for numerical integration.

$$V_{new} = V + dt * a \qquad (3\text{-}7)$$

Where dt is the time increment in seconds between subsequent calls on the physics engine.

The car's position is in turn determined by integrating the velocity over time.

$$P_{new} = P + dt * v \qquad (3\text{-}8)$$

With these forces, we can simulate car acceleration fairly accurately. Together they also determined the top speed of the car since as the velocity of the car increases the resistance forces also increases. At some point the resistance forces and the engine force cancel each other out and the car has reached its top speed.

Chapter 4: Engine Torque

When the engine runs, it generates a certain amount of torque. The torque that an engine delivers depend on the speed at which the engine is turning, usually expressed in terms of revolutions per minute, or *rpm*. The relationship torque versus rpm is usually provided as a curve known as a torque curve. A torque curve of the 2004 Porsche Boxster S is shown in Figure 4-1.

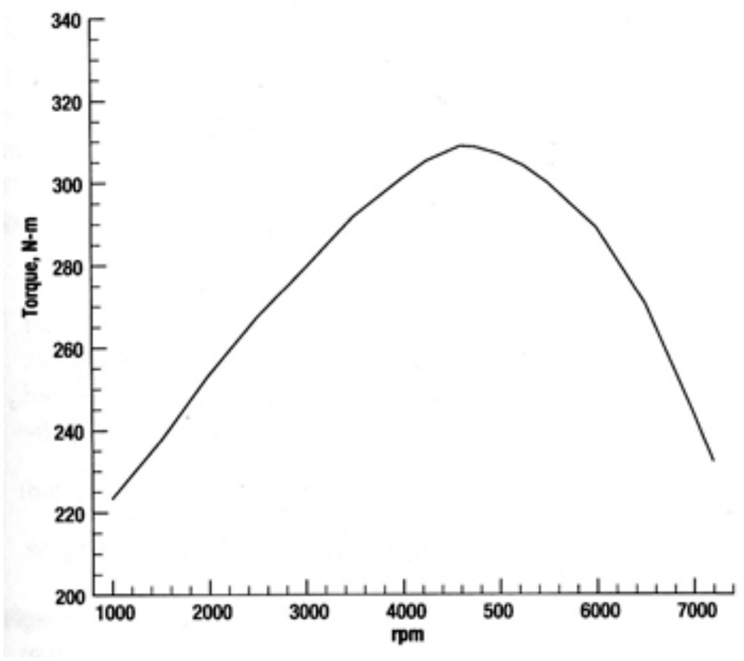


Figure 4-1. Torque curve for the 2004 Boxster S

The figure shows the maximum torque the engine can deliver at a given rpm. The actual torques that engine delivers depends on the throttle position and is a fraction between 0 and 1. The curve is defined in a specific range since any higher (above the so-called "redline") will damage the engine.

The angular velocity of the engine in rad/s is obtained by multiplying the engine turnover rate by 2π and dividing by 60.

$$\omega_e = 2\pi\ \Omega_e / 60 \tag{4-1}$$

For game programming purposes, we can approximate the torque curve as a series of straight lines. The torque curve for the Boxster S as shown in figure 3-1, for example, can be approximate by three straight lines.

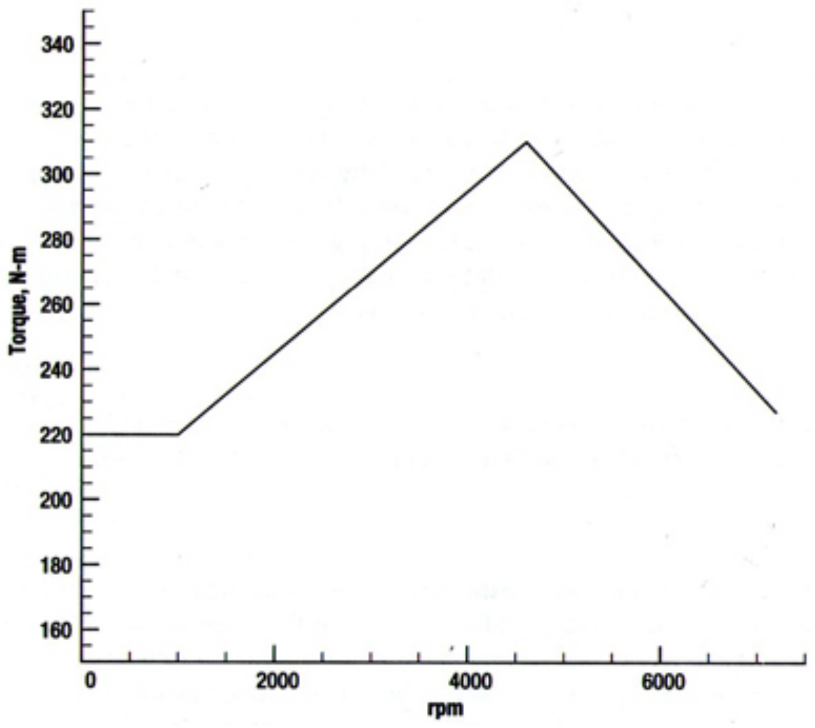


Figure 4-2. Simplified torque curve for the 2004 Boxster S

4.1 Gears and Wheel Torque

The torque applied to the wheels is not the same as the engine torque because the engine torque passes through a transmission before it is applied to the wheels. The gear ratio between two gears is the ratio of the gear diameters. Car transmission will typically have between three and six forward gears and one reverse gear. There is also an additional set of gears between the transmission and the wheels known as the differential and the gear ratio of this gearset is called the final drive ratio. The wheel toque can be obtained using the following equation.

$$T_w = T_e * g_k * G \tag{4-2}$$

Where T_e is the engine torque, g_k is the gear ratio of whatever gear the car is in and G is the final drive ratio.

The relationship between the engine turnover rate and the wheel angular velocity is as follows.

$$\omega_w = 2\pi\ \Omega_{e\ /\ (}60*g_k*G) \tag{4-3}$$

If we assume that the tires roll on the ground without slipping, the translational velocity of the car can be related to the engine turnover rate as follows.

$$V = r_w\omega_w = r_{w\ *}\ 2\pi\ \Omega_{e\ /\ (}60*g_k*G) \tag{4-4}$$

4.2 Gear Shifting and Automatic transmission

Lower gear results in a greater acceleration but gear shifting is necessary as the velocity of the car is a function of the engine turn over rate and gear ratios, and every car has a limit to how fast the engine can turn over. There are two types of transmission – manual transmission and automatic transmission. To simulate automatic transmission in the game, we could just specify an engine turnover rate in which the transmission would shift.

Chapter 5: Cornering

Turning at low speeds, the wheel normally rolls in the direction they are pointed at. In high-speed turns, the wheels can be heading in one direction while their movement is still in another direction. This means there is a velocity component that is at right angle to the wheels causing them to being pushed sideways. The angle between the car's orientation and the car's velocity vector is known as the side slip angle (*beta*).

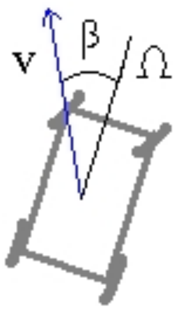


Figure 5-1. Side slip angle (*beta*).

In high speed cornering, the tires develop lateral forces also known as the cornering force. This force depends on the slip angle (alpha), which is the angle between the tire's heading and its direction of travel, and the weight on that tire.

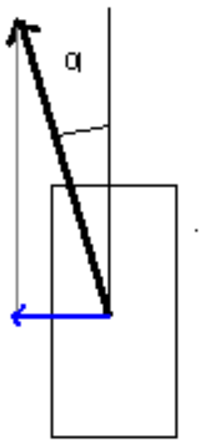


Figure 5-2. Slip angle (*alpha*).

At low slip angles, the cornering force can be determined as follows.

$$F_{cornering} = C_a * alpha \tag{5-1}$$

Where C_a is known as the cornering stiffness.

The slip angles for the front wheels and rear wheels are given by the following equations.

$$\alpha_{front} = \arctan((V_{lat} + \omega * b) / V_{long})) – \sigma * sgn(V_{long})$$
$$\alpha_{front} = \arctan((V_{lat} - \omega * c) / V_{long})) \tag{5-2}$$

Where ω is the angular speed of the car, σ is the steering angle, V_{lat} is the velocity in the lateral direction, V_{long} is the velocity in the longitudinal direction, b is the distance from CG to the front axle, c is the distance from CG to the rear axle, and c is the sign function that extracts the sign of a real number.

For high slip angles, the relationship between slip angle and cornering force is not linear but described by curve diagrams such as the following.

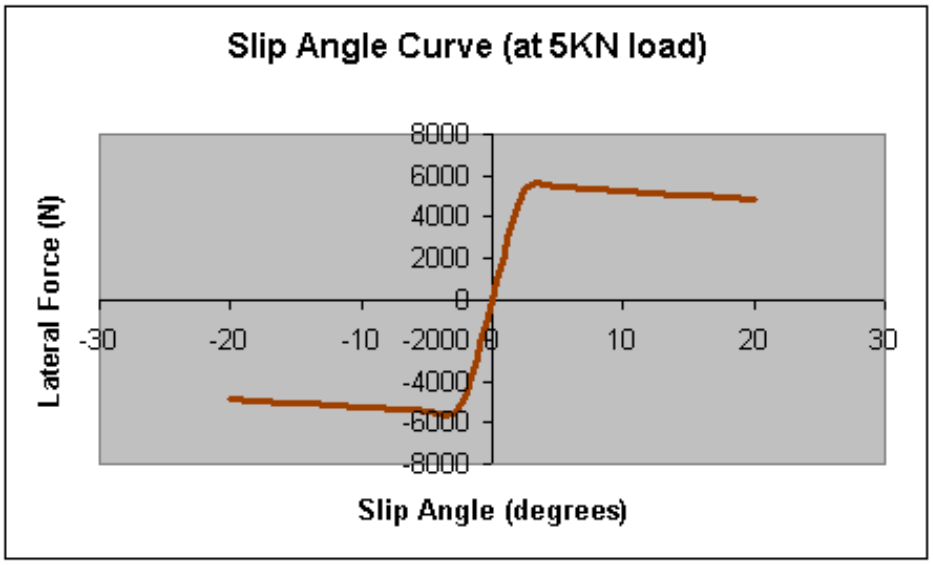


Figure 5-3. Slip angle VS lateral force

The lateral force depends on the load of the tire. In order to find lateral forces at different loads, we could assume that the lateral force is linear with load and create a normalized lateral force diagram by dividing the lateral force by 5KN. Lateral force can, therefore, be determined as follows.

$F_{lat} = C_a * alpha$

F_{lat} = Capped F_{lat} to a defined maximum normalized friction force (diameter of friction circle)

$F_{lat} = F_{lat} * load$

$F_{lat} = F_{lat} * 0.5$ (in case the wheel slip) (5-3)

The lateral force also introduce a torque which causes the car body to turn, which can be calculated as follows.

$Torque = \cos(\sigma) * F_{lat,\ front} * b - F_{lat,\ rear} * c$ (5-4)

Where σ is the steering angle, b is the distance from CG to the front axle, and c is the distance from CG to the rear axle.

Dividing the torque by the inertia, we obtain the angular acceleration. Integrating the angular acceleration over time, we get the angular velocity. We then integrate the angular velocity again to obtain the angular position of the car.

Chapter 6: Implementing car physics engine (part 1)

This chapter demonstrates how to implement car physics engine using the knowledge from chapter 3-5 (ignoring the suspension system and the height map for a moment). The pseudo code of the engine is as follows.

```
1. Transform velocity in world reference frame to velocity in car reference frame ($V_x = V_{long}$, $V_z = Vl_{ong}$). Convention for car reference frame: x – pointing to the front, z – pointing to the right
2. Compute the slip angles for front and rear wheels (equation 5.2)
3. Compute $F_{lat} = C_a$ * slip angle (do for both rear and front wheels)
4. Cap $F_{lat}$ to maximum normalized frictional force (do for both rear and front wheels)
5. Multiply $F_{lat}$ by the load (do for both rear and front wheels) to obtain the cornering forces.
6. Compute the engine turn over rate $\Omega_e = V_x\ 60 * g_k * G / (2\pi * r_w)$
7. Clamp the engine turn over rate from 6 to the defined redline
8. If use automatic transmission call automaticTransmission() function to shift the gear
9. Compute the constant that define the torque curve line from the engine turn over rate
10. From 9, compute the maximum engine torque, $T_e$
11. Compute the maximum torque applied to the wheel $T_w = T_e * g_k * G$
12. Multiply the maximum torque with the fraction of the throttle position to get the actual torque applied to the wheel ($F_{traction}$ - The traction force)
```

13. If the player is braking replace the traction force from 12 to a defined braking force
14. If the car is in reverse gear replace the traction force from 12 to a defined reverse force
15. Compute rolling resistance $F_{rr,\,x} = -\ C_{rr} * V_x$ and $F_{rr,z} = -\ C_{rr} * V_z$
16. Compute drag resistance $F_{drag,\,x} = -\ C_{drag} * V_x * |V_x|$ and $F_{drag,\,z} = -\ C_{drag} * V_z * |V_z|$
17. Compute total resistance ($F_{resistance}$) = rolling resistance + drag resistance
18. Sum the force on the car body

    $F_x = F_{traction} + F_{lat,\,front} * \sin(\sigma) * F_{resistance,\,x}$

    $F_z = F_{lat,\,rear} + F_{lat,\,front} * \cos(\sigma) * F_{resistance,\,z}$
19. Compute the torque on the car body

    $Torque = \cos(\sigma) * F_{lat,\,front} * b - F_{lat,\,rear} * c$
20. Compute the acceleration

    $a = F / M$
21. Compute the angular acceleration

    $\alpha = Torque/Inertia$
22. Transform the acceleration from car reference frame to world reference frame
23. Integrate the acceleration to get the velocity (in world reference frame)

    $V_{wc} \mathrel{+}= dt * a$
24. Integrate the velocity to get the new position in world coordinate

    $P_{wc} \mathrel{+}= dt * V_{wc}$
25. Move the camera to follow the car.

26. Integrate the angular acceleration to get the angular velocity

    ω += dt * α

27. Integrate the angular velocity to get the angular orientation

    Yaw angle += dt * ω

28. Obtain the rotation rate of the wheels by dividing the car speed with the wheel radius

    Wheel rotation rate = car speed / wheel radius

29. Re-render the car

Car Physics Engine (Part 1)

In sum, the physics engine first transforms the world reference frame to velocity in car reference frame. It then computes the force and torque on the car body. Finally, it computes the new position and the new angular orientation of the car and re-render the car.

The pseudo code of automatic transmission function is as follows.

```
1. Define the rpm when the transmission would shift
2. Compute the maximum velocity for each gear before the gear shift
        $V_{max} = r_{wheel} * 2 \pi * \Omega_e / (60 * g_k * G)$
3. If $V_x$ is more than the maximum velocity of the current gear, shift the
   gear up and return
4. If the car is in gear one, do nothing and return
5. If $V_x$ is less than the maximum velocity of the lower gear (maximum
   velocity of (current gear -1)), shift the gear down and return
```

Automatic Transmission

Chapter 7: Weight Transfer and Suspension System

7.1 Weight Transfer

An important effect that happens during a turn is that weight is shifted to the outside wheel. The effect of dynamic weight transfer also happens when accelerating or braking. When braking hard the car will nosedive. During acceleration, the car leans back. The effect of weight transfer dramatically improves the visual effects and adds a lot of realism to driving games. The weight distribution also affects the cornering force as shown in equation 5-3 of chapter 5. Additionally, it affects the compression of the suspension system. For a stationary vehicle, the total weight of the car is distributed over the front and rear wheels as follows.

$$W_f = (c / L) * W$$
$$W_r = (b / L) * W \qquad (7\text{-}1)$$

Where b is the distance from CG to front axle, c is the distance from CG to rear axle, and L is the wheelbase (b + c)

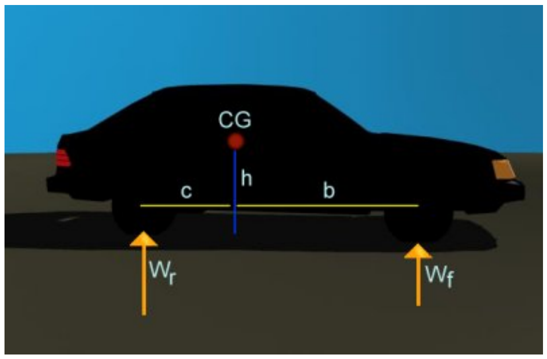


Figure 7-1. The distribution of the weight of the car

During acceleration the weight of the car is thrown backwards. This cause the front wheels have less weight to bear.

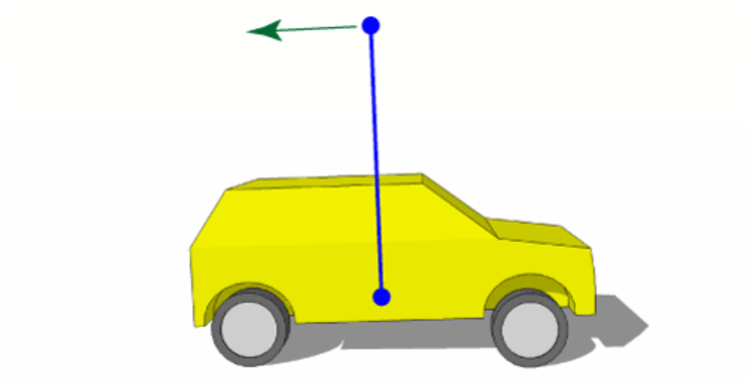


Figure 7-2. Acceleration – rear weight transfer

Conversely, forward weight transfer occurs during deceleration.

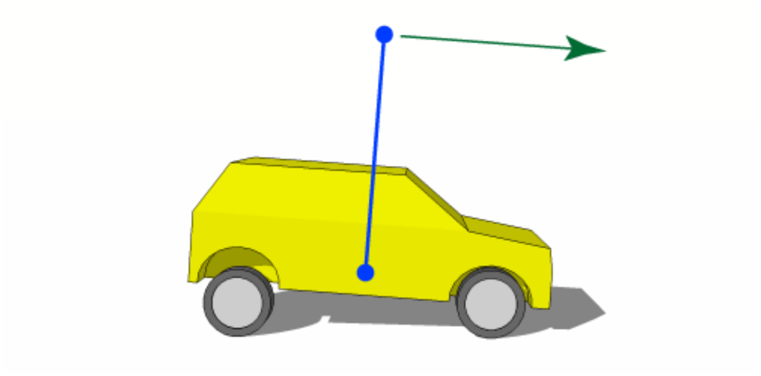


Figure 7-3. Deceleration – forward weight transfer

The formula to determine the weight transfer as a result of lateral force is as follows.

$$\text{Weight transfer} = (h / L) * M * |a| \qquad (7\text{-}2)$$

Where h is the height of the CG, M is the car's mass, and a is the acceleration.

When a car is cornering at speed, the car's weight transfer from inside wheel to the outside wheel. There is weight transfer due to lateral acceleration as well as weight transfer introduced by body roll. The weight transfer due to lateral acceleration is determined as follows.

Weight transfer = (Lateral acceleration / g) * weight * Height of CG / Track width (7-3)

Body rolls also introduce weight transfer thus reduction of total grip. The following drawing shows a car's body roll when cornering. (The outside wheels are on the left and the inside wheels are on the right.)

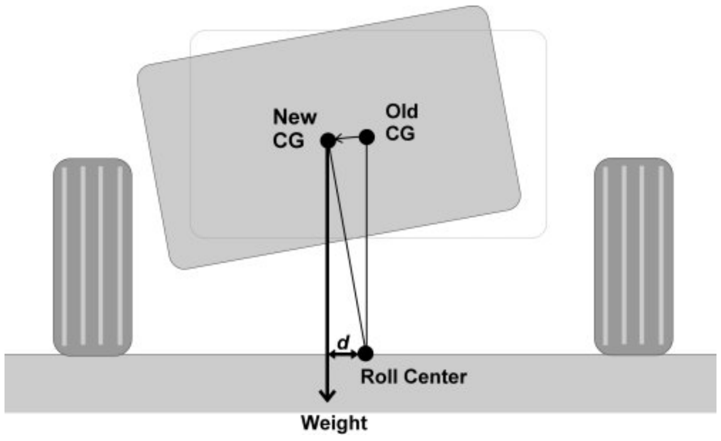


Figure 7-4. Body roll due to cornering

The amount of weight transfer due to body roll is determined as follows.

Weight transfer = (Height of CG * sin (body roll angle)/ Track width) * weight (7-4)

Although the weight transfer due to body roll is not a great amount compare with the weight transfer due to lateral acceleration especially for car with long track width, its influence should not be ignored.

There is another weight transfer that occurs when a car is moving on an incline terrain or a road that has noises. In this case, we could use equation 6-4 for weight transfer in the lateral direction as an approximation. For weight transfer in the longitudinal direction, it is determined as follows.

Weight transfer = (Height of CG * sin (body roll angle)/ Car length) * weight (7-5)

7.2 Suspension System

7.2.1 Springs

Suspension is a system of springs, shock absorber, and linkages that connects a car to its wheels and allows relative motion between the two. Implementing the suspension system adds a lot of realism to the dynamics of the car. The suspension system of a car is very complicated. For game programming purposes, we could just assume that a spring connects each wheel to the car body as shown in the figure 7-5.

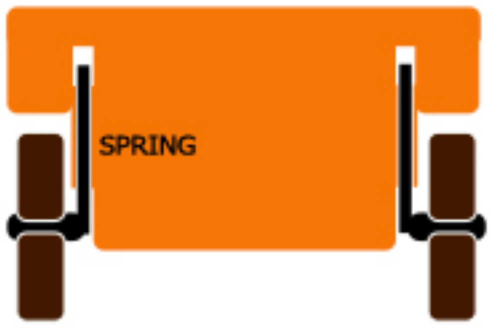


Figure 7-5. A simple suspension system

The springs apply equal and opposite force to each object. It force follows Hook's law and is a function of the stretched or compressed length of the spring relative to the rest length of the spring and the spring constant of the spring. The spring constant is a quantity that relates the force exerted by the spring to its deflection. The spring equation of Hook's law is as follows.

$$Fs = Ks\,(L - r) \qquad (7\text{-}6)$$

Where Fs is the spring force, Ks is the spring constant, L is the stretched or compressed length of the spring, and r is the rest length of the spring.
The force exerted on each wheels can be determined using equations in section 7.1

7.2.2 Linkages

The system of linkages of a suspension keeps the vehicle occupant comfortable and well isolated from road noise, bump, and variations, etc. For game programming purposes, we could just assume that a moveable link connects each wheel to the spring as shown in the figure 7-6.

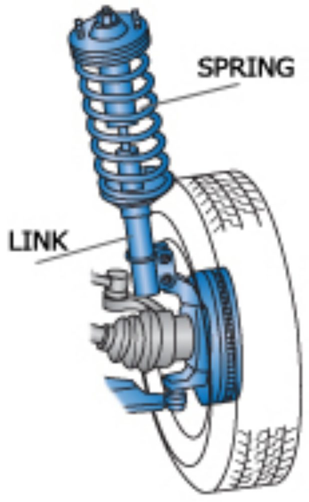


Figure 7-6. A link connects a wheel to a spring

To simulate the dynamics of the linkages, first we determine the surface of the terrain with the road noise removed. After that, we determine the height of the road noise affecting each wheel. Finally, we move the links according to these heights. But they will not be moved upward more than the specified value. In this way, the orientation of the car's body is changing minimally. Figure 7-7 illustrates the algorithm of simulating the dynamics.

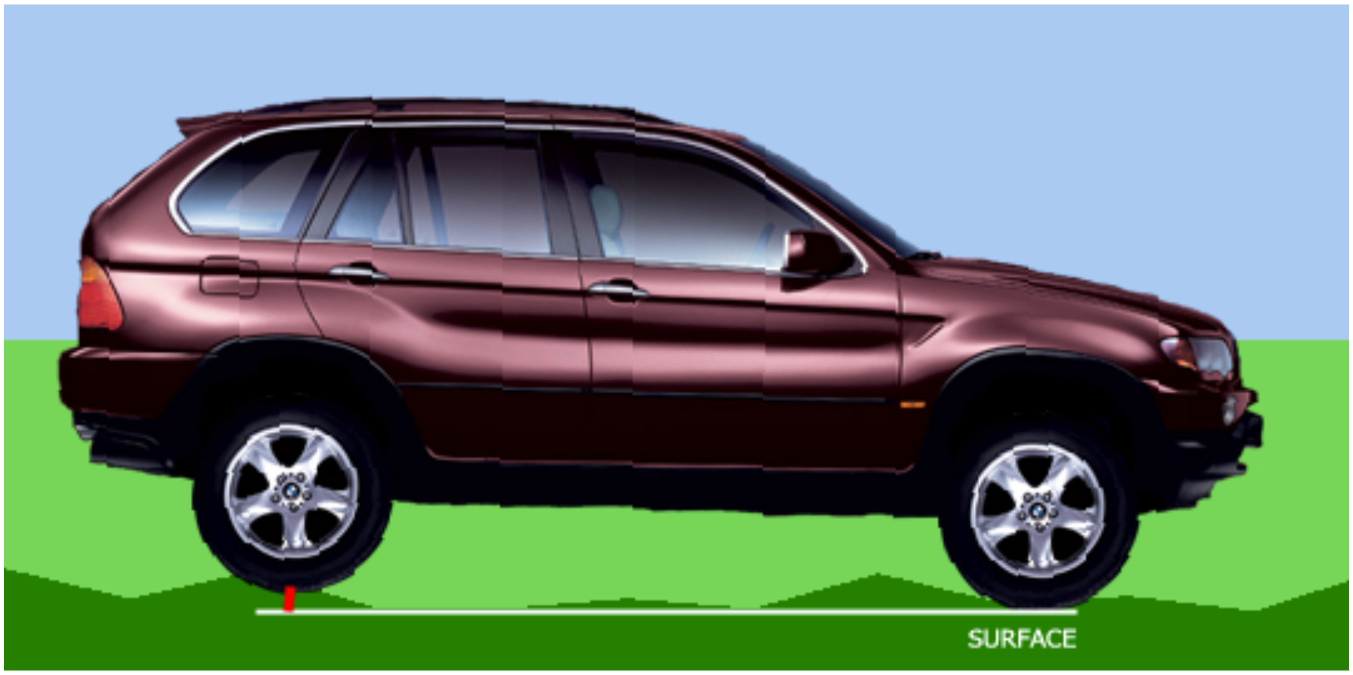


Figure 7-7. The dynamics of a linkages

In the figure, the white line shows the surface of the terrain with the road noise removed and the red line shows the height of the road noise. In this case, the rear wheels will be moved upward to the body of the car in order to maintain orientation of the car's body as much as possible.

7.2.3 Camber angle

Camber angle is the angle that the vertical axis of the wheel makes with the road surface. Negative camber is when the top of the wheel leans to the centre of the car. Positive camber is when the top of the wheel leans out. Considerable changes in camber angle are effect under condition of accelerating and braking. On acceleration or braking, the suspension produces positive camber to the undriven wheels and negative camber to the driven wheels as shown in figure 7-8. The magnitude of the camber angle depends on the magnitude of the car's acceleration. Under constant velocity, the wheels tend toward their static position. For game programming purposes, we could defined a maximum camber angle and compute the current camber from the percentage of the longitudinal weight transfer times the maximum camber angle.

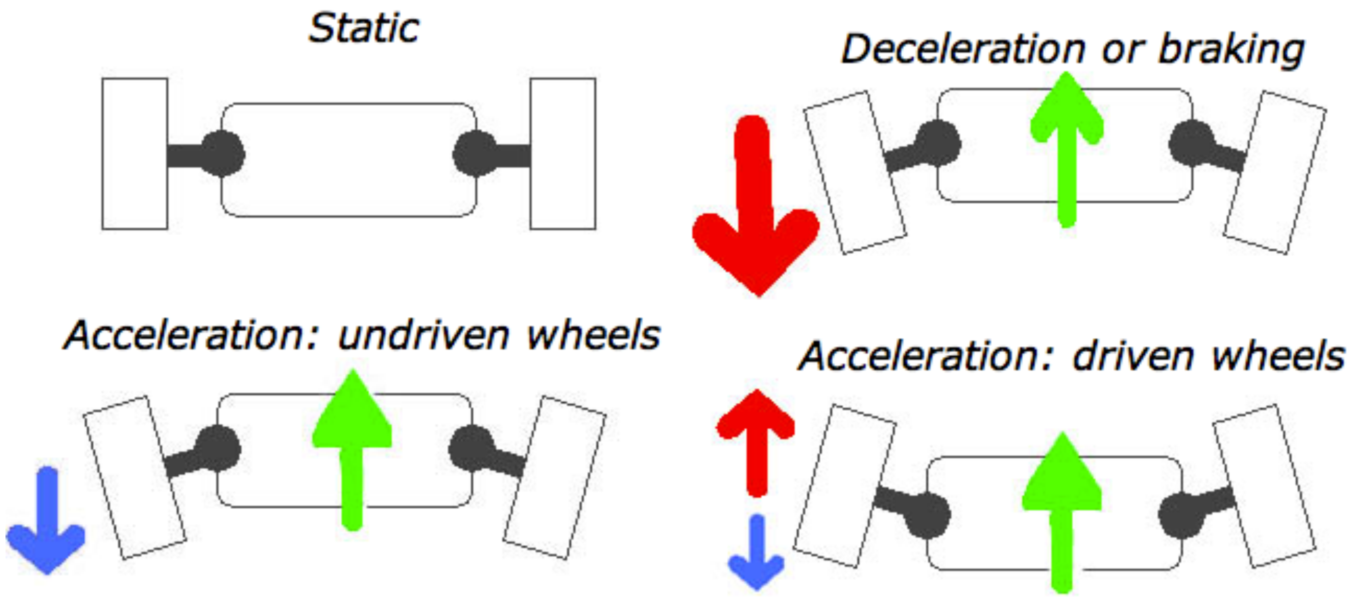


Figure 7-8. Camber angle and chassis dynamics

7.2.4 Different types of terrain

A racing game usually comprises more than one type of terrain. To simulate types of terrain in the car physics engine, we can specify the region for each type of terrain. A type of terrain has two attributes, which are rolling resistance coefficient and maximum height variation. For road, the maximum height variation is set as zero and the rolling resistance coefficient is set to a low value, whereas for tracks made of mud, the maximum height variation is set to a low value and the rolling resistance coefficient is set to a high value. The car moves slower in terrain where the rolling resistance coefficient is set to a higher value. The maximum height variation controls the roughness of the road surface. The car physics engine generates a random number between zero and the maximum height variation and add this number to the height that a wheel lies on. It then repeats this process for the three other wheels. In this way, it simulates the roughness of the terrain that the car moves on.

Chapter 8: Implementing car physics engine (part 2)

This chapter demonstrates how to implement car suspension using the knowledge from chapter 7. The pseudo code of the engine is as follows.

```
1. Compute weight on front and rear axle
       $W_{frontAxle} = (c / L) * Weight$
       $W_{rearAxle} = (b / L) * Weight$
2. Compute weight transfer due to acceleration
       $W_{transfer, acc} = (h / L) * M * a_x$
3. Update weight on front and rear axle
       $W_{front} += W_{transfer, acc}$
       $W_{rear} -= W_{transfer, acc}$
4. Compute weight transfer due to lateral force
       $W_{transfer, lat, rear} = (a_z / G) * W_{rear} * h / Track width$
       $W_{transfer, lat, front} = (a_z / G) * W_{front} * h / Track width$
       $W_{transfer, lat} = W_{transfer, lat, rear} + W_{transfer, lat, front}$
5. Update weight on each wheel
       $W_{rear, right} -= W_{transfer, lat, rear}$
       $W_{rear, left} += W_{transfer, lat, rear}$
       $W_{front, rear} -= W_{transfer, lat, front}$
       $W_{front, left} += W_{transfer, lat, front}$
6. Compute the body roll angle
       Body roll angle = $sgn(a_z) * (W_{transfer, lat} / Weight) * max body roll angle$
7. Compute the camber angle
       Camber angle = $| (W_{transfer, long} / Weight) * max camber angle |$
```

```
8.  Compute weight transfer due to body roll
        $W_{transfer, roll} = (sgn(a_z) * h * sin(|roll angle|) / track width) * weight$
9.  Update weight on each wheel
        $W_{rear, right}$ -= $W_{transfer, roll}$ / 2
        $W_{rear, left}$ += $W_{transfer, roll}$ / 2
        $W_{front, rear}$ -= $W_{transfer, roll}$ / 2
        $W_{front, left}$ += $W_{transfer, roll}$ / 2
10. Compute the position of the bottom of the wheels relative to CG
11. Compute the orientation vectors from the vectors in 10
            RL_FR = normalize (FR_wheelPos – RL_wheelPos)
            RR_FL = normalize (FL_wheelPos – RR_wheelPos)
            RL_FL= normalize (FL_wheelPos – RL_wheelPos)
            RL_RR = normalize (RR_wheelPos – RL_wheelPos)
            RR_FR = normalize (FR_wheelPos – RR_wheelPos)
12. Rotate the vectors in 10 by the yaw angle and add the CG
    position to get the wheels bottom positions
13. Update the bottom position of each wheel by height value from
    the height map
```

```
14. Compute the roll angle both longitudinal (x) and lateral (z)
        $dx = max(maxRear-minFront,minRear-maxFront)$
        $dz = max(maxRight-minLeft,minLeft-maxRight)$
        $angleX = arcsin(|dx|/ car length)$
        $angleZ = arcsin(|dz|/ track width)$
    Where maxRear is the maximum height of the rear wheels
    maxFront is the maximum height of the front wheels
    maxRight is the maximum height of the right wheels
    maxLeft is the maximum height of the left wheels
    minRear is the minimum height of the rear wheels
    minFront is the minimum height of the front wheels
    minRight is the minimum height of the right wheels
    minLeft is the minimum height of the left wheels
15. Compute the weight transfer in the lateral and longitudinal
  direction
    $Weight_{TransferX} = (sgn(dx) * h * sin(angleX) / car length)*weight$
    $Weight_{TransferZ} = (sgn(dz) * h * sin(angleX) / car width)*weight$
16. Update weight on each wheel
    $Weight_{frontAxle}$ += $Weight_{TransferX}$
    $Weight_{rearAxle}$ -= $Weight_{TransferX}$
    $W_{rear, right}$ -= $W_{transfer, x} / 2 + W_{transfer, z} / 2$
    $W_{front, right}$ += $W_{transfer, x} / 2 + W_{transfer, z} / 2$
    $W_{front, left}$ += $W_{transfer, x} / 2 + W_{transfer, z} / 2$
    $W_{rear, left}$ -= $W_{transfer, x} / 2 + W_{transfer, z} / 2$
```

```
17. Compute the car body positions
        RR_bodyPos = RR_wheelPos + vector (0,lengthRear0, 0)
        FR_bodyPos = FR_wheelPos + vector (0,lengthFront0, 0)
        FL_bodyPos = FL_wheelPos + vector (0,lengthFront0, 0)
        RL_bodyPos = RL_wheelPos + vector (0,lengthRear0, 0)
        Where lengthRear0 and lengthFront0 are the lest length of the
    spring of the rear and front wheels respectively
18. Compute the distance that the springs are compressed
        $\text{Spring length}_{\text{rear, right}} = W_{\text{rear, right}} / K_{\text{rear}}$
        $\text{Spring length}_{\text{front, right}} = W_{\text{front, right}} / K_{\text{front}}$
        $\text{Spring length}_{\text{front, left}} = W_{\text{front, left}} / K_{\text{front}}$
        $\text{Spring length}_{\text{rear, left}} = W_{\text{rear, left}} / K_{\text{rear}}$
19. From the compressed length of the springs in 18, update the
    positions of the car body
        RR_bodyPos.y -= $\text{Spring length}_{\text{rear, right}}$
        FR_bodyPos.y -= $\text{Spring length}_{\text{front, right}}$
        FL_bodyPos.y -= $\text{Spring length}_{\text{front, length}}$
        RL_bodyPos.y -= $\text{Spring length}_{\text{rear, left}}$
20. Compute the road noise and update the positions of the car body
    accordingly.
21. Compute the positions of the car body relative to CM
        RR_bodyRel = RR_bodyPos – CM_Pos
        FR_bodyRel = FR_bodyPos – CM_Pos
        FL_bodyRel = FL_bodyPos – CM_Pos
        RL_bodyRel = RL_bodyPos – CM_Pos
```

```
22.   From positions of the car body in 21, compute the orientation
      vectors.
23.   From the orientation vectors in 11 and the orientation vectors in
      22, compute the transformation matrix using the function
      matrixFrom2Vector()
23.   Re-render the car
```

In sum, the physics engine first determines the weight distribution on each wheel. It then computes the length of the compressed springs and the positions of the car body. Finally, it computes the transformation matrix of the car and passes it to the render module to re-render the car.

The pseudo code of the matrixFrom2Vector() function, which computes the rotation matrix from 2 vectors, is as follows.

```
1. matrixFrom2Vector(a, b) // a = source vector, b = destination vector
2. Use a x b as the rotation axis
3. Derive cosine of the rotation angle, c, from the dot product of a and b
4. Derive sine of the rotation angle, s, from 1- c^2
5. If C is 1 or -1 return the identity matrix
6. Compute t = 1- c
7. Compute x = normalized axis x coordinate
8. Compute y = normalized axis y coordinate
9. Compute z = normalized axis z coordinate
10. Compute the transformation matrix
t*x*x + c       t*x*y - z*s     t*x*z + y*s
t*x*y + z*s     t*y*y + c       t*y*z - x*s
t*x*z - y*s     t*y*z + x*s      t*z*z + c
11. Return the transformation matrix
```

matrixFrom2Vector

The car is divided into 5 parts, the body and the four wheels; each has its own transformation matrix. The pseudo code of the getBodyMatrix() function, which computes the transformation of the car body, is as follows.

```
1. Roll the car according to the body roll angle (around x-axis)
2. Then rotate the car according to the yaw angle (around y-axis)
3. Then multiply the output by the rotation matrix from the suspension
   system
4. Finally translate the car to the CM position and return the final
   transformation matrix
```

getBodyMatrix()

The pseudo code of the getRearRightWheelMatrix() function, which computes the transformation of the rear right wheel matrix, is as follows.

```
1. Revolve the wheel (around z-axis)
2. Then rotate the wheel according to the camber angle (around x-axis)
3. Then rotate the wheel according to the yaw angle of the car  (around y-axis)
4. Then multiply the output by the rotation matrix from the suspension system
5. Then translate the car to the CM position
6. From the compressed spring length information, compute the vector from CM to the wheel and named it relToVec
7. Rotate relToVec according to the yaw angle of the car
8. Multiply relToVec by the rotation matrix from the suspension system
9. Create a translation matrix of relToVec and named it rotMatrix_translate_rel_car
10. Multiply the output by rotMatrix_translate_rel_car and return it as the final transformation matrix
```

The rotation matrices of the other wheels is determined is a similar fashion

The pseudo code of the chassis() function, which simulates the dynamics of the linkages due to road noise, is as follows.

```
1. From the position of the four wheels of the car and the data from the height map, compute the vector on the terrain in the lateral direction of the car with the most gradual slope.
2. Repeat 1 but for the longitudinal direction.
3. From 1 and 2, construct a plane and compute the height of the road noise affecting the wheels.
4. Move the links according to the height of the road noise.
5. Clamp the movement of the link to the defined value.
```

Chassis()

Chapter 9: Projectiles and collision response from wall

In chapter 8, we ignore collision detection with wall. To check the collision between the car and a wall we have to obtain the height of the terrain that the four wheels lie on. If the slope of the terrain that one of the four wheels lie on is higher than a defined threshold and the maximum height difference of the terrain that the wheels lies on is higher than a defined threshold, then the car collide with the wall. If the car collides with the wall, the velocity is multiplied by a negative value as shown in equation 9-1 before determining the position of the car by integrating the velocity.

$$\text{Velocity (in world reference)} \mathrel{*}= -0.8 \tag{9-1}$$

In a certain type of terrain, using the suspension system in chapter 8, the car body may cut through the terrain as shown in figure 9-1. This is because some of the four wheels should not lie on the ground.

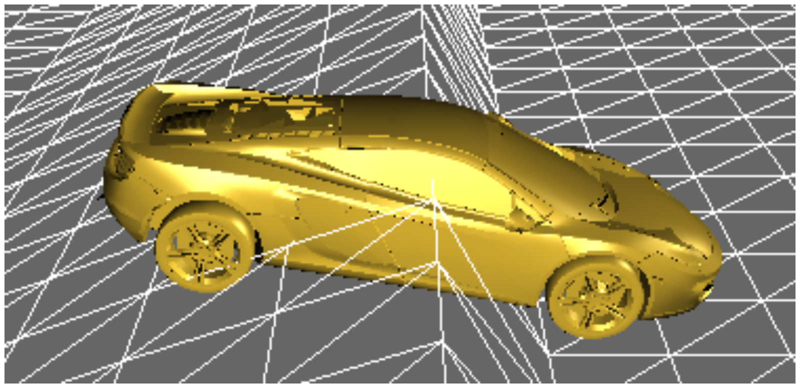


Figure 9-1. Car body cut through the terrain

To fix this issue, we find the highest point on the terrain from the rear left wheel to the front left wheel (highest left) and the highest point of the terrain from the rear right wheel to the front right wheel (highest right). We draw a vector from the rear left wheel to the highest left and another vector from the rear right wheel to the highest right and choose the steeper as the orientation vector. From the orientation vector, we could compute the appropriate transformation matrix using the same algorithm as shown in chapter 8. Thus, the corrected car orientation is shown in figure 9-2.

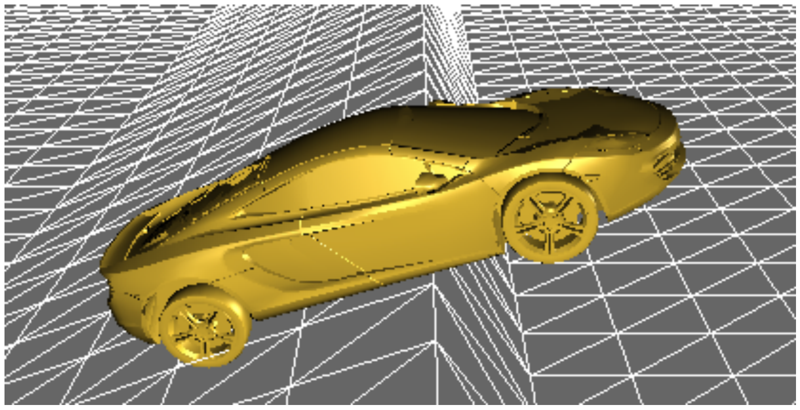


Figure 9-2. Corrected car body orientation

The easiest way to determine when the car is travelling through the air in a projectile motion is to keep track the orientation of the body and if the front of the car drop to a defined angle compare to the front of the car in the previous frame, then the car starts to travel in a projectile motion. If the speed of the car is higher than a defined value, then use the orientation of the car in the previous frame; otherwise use the orientation of the car in the current frame. Set the current velocity of the car as the initial velocity of the car.

To update the position of the car in a projectile motion, the x and z components of the car's velocity is assume to be constant. The y component of the car's velocity is determined using equation 9-2.

$$\text{Velocity}_{Y} \mathrel{+}= \text{time step} * - g \tag{9-2}$$

Where g is the local gravitational field ≈ 9.8

To determine the orientation of the car in a projectile motion, first we compute the moment of force around the car's center of mass from the weight on the four wheels. The moment of force in the longitudinal and lateral directions are handled separately. Once we have the moment of force, we could determine the angular acceleration, and thus the transformation matrix. The car stops travelling in a projectile motion if one of the wheels touches the ground.

Chapter 10: Implementing Collision Response

This chapter addresses what happens when two cars run into each other. For game applications, speed is a major issue, and very accurate collision detection can be slow. For the sake of speed and simplicity, a bounding sphere scheme along with bounding box and vertex edge and vertex face collision detection schemes are used in this master project. Here, bodies that are colliding are treated as rigid irrespective of their construction and material. There is a method to deal with collision response known as penalty method. In penalty methods the force at impact is represented by a temporary spring that gets compressed between the objects at the point of impact. This spring compresses over a very short time and applies equal and opposite forces to the colliding bodies to simulate collision response. However, this method is not as accurate as the analytic method. Therefore, the treatment of collision response in this project is based on classical Newtonian impact principles. By formulating and solving Newtonian dynamical equations, we will end up with a formula for J that takes into account both linear and angular effects, which you can then use to find the linear and angular velocities of each body immediately after impact as follows.

$$\begin{aligned}
&J = -V_r\,(e+1)/\{1/m_1 + 1/m_2 + n \text{ dot } [(r_1 \text{ x } n)/I_1] \text{ x } r_1 + n \text{ dot } [(r_2 \text{ x } n)/I_2] \text{ x } r_2\} \\
&V_{1+} = V_{1-} + (Jn)/m_1 \\
&V_{2+} = V_{2-} + (-Jn)/m_2 \\
&\omega_{1+} = \omega_{1-} + (r_1 \text{ x } Jn) / I_{cg} \\
&\omega_{2+} = \omega_{2-} + (r_2 \text{ x } -Jn) / I_{cg} \qquad (10\text{-}1)
\end{aligned}$$

Where V_r is the relative velocity along the line of action at the impact point p, n is a unit vector along the line of action at the impact point pointing out from body 1, e is the coefficient of restitution, I is the moment of inertia tensor, r is the vector from the body's center of gravity to the impact point, m refers to mass, v refers to velocity subscript 1 refers to body 1, subscripts 2 refers to body 2, subscript – refers to the instant just before impact, and subscripts + refers to the instant just after impact.

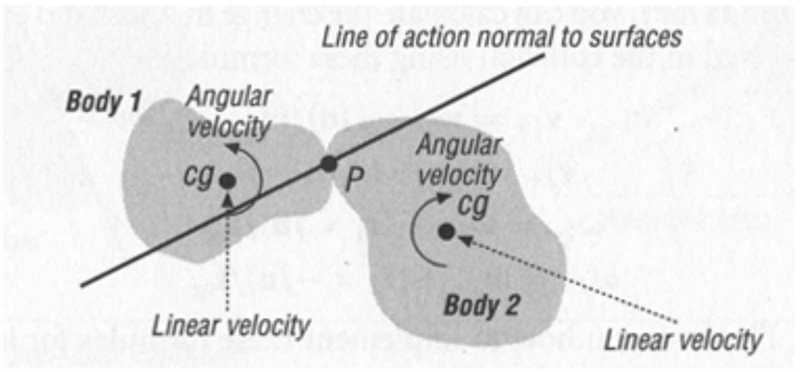


Figure 10-1. Two colliding cars

To determine whether or not there is a collision, we need to consider two factors. First, we need to check whether they are close enough, within numerical tolerances, to be considered in colliding contact. Then we have to do another check on the relative normal velocity. A collision occurs when they are in contact and the points are moving toward each other. The types of contact that will be checked for are vertex-vertex and vertex-edge contact as shown in figure 10-2. In edge-vertex collisions the normal is perpendicular to the edge that is involved in the collision. In vertex-vertex collisions, however, the normal is ambiguous, so we could approximate using the line connecting the centers of gravity of the car as the normal.

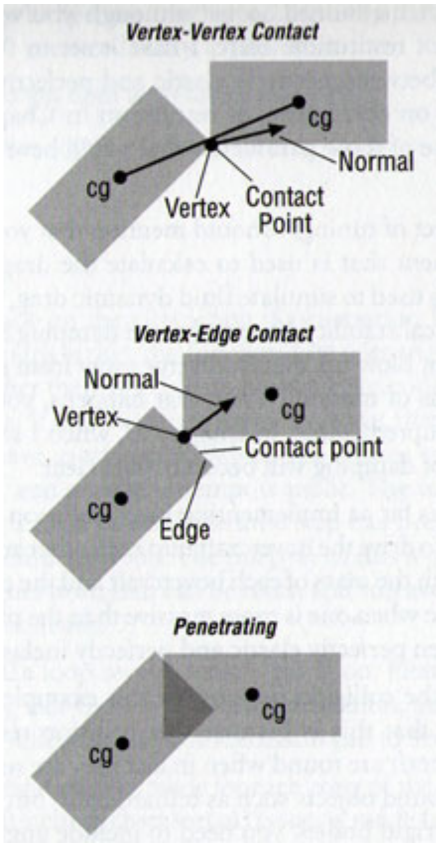


Figure 10-2. Types of Collision

The pseudo code for the collision response module is as follows.

```
1. Compute the vector from the CM of the car to the center of the car
   (before multiplying the transformation matrix).
2. Multiply the vector from 1 by the transformation matrix and add the
   position of the center of the car to get the center of the bounding
   sphere.
3. Compute the radius of the bounding sphere, which is half the length
   of the car.
4. Compute the position of the bounding box vertices relative to the CM
   of the car (before multiplying the transformation matrix).
5. Multiply the vectors from 4 by the transformation matrix and add the
   position of the center of the car to get the vertices of the bounding
   box.
```

```
6.  Compute the edges of the bounding box from the vertices of the
    bounding box.
7.  Repeat process 1-6 for another car.
8.  If the bounding circles are not colliding, exit the module.
9.  Check for vertex-vertex collisions, by iterating through each vertex in
    the list of the first car, comparing it each vertex in the list of another
    car.
10. Begin vertex-vertex loop
11. Compute the collision normal vector
    collisionNormal = (car1.CMPosition – car2.CMPosition). normalize()
12. Compute the collision point relative to CM
        Car1.collisionPoint = CollisionPoint – car1.CMPosition
        Car2.collisionPoint = CollisionPoint – car2.CMPosition
13. Compute the velocity at the point of collision, which is the sum of the
    velocity of the CM and the tangential velocity.
        V1 = Car1.CMVelocity + Car1.angularVelocity X
        Car1.collisionPoint
        V2 = Car2.CMVelocity + Car2.angularVelocity X
        Car2.collisionPoint
14. Compute the relative velocity Vr = V1 – V2
15. If the points are within a specified distance from each other and they
    are moving toward each other (dot product of Vr and collisionNormal <
    0), then a collision occurs.
16. End vertex-vertex loop
17. If vertex-vertex check fails, then check for vertex–edge collision by
    iterating through each vertex in the list of the first car to see whether
    it is in contact with each edge in the list of another car.
```

```
18. Begin vertex-edge loop. Figure 10-2 illustrates the geometry.
19. Compute the unit vector pointing along the edge.
        u = (edge vertex 2 – edge vertex 1).normalize()
20. Compute the vector from the first vertex on the edge to the vertex under consideration.
        p = vertexList1[i] – edge vertex 1
21. Compute the distance from the first edge vertex, along the edge, to the point upon which the vertex projects.
        proj = p.dot(u) * u
22. Compute the minimum distance from the vertex to the edge.
        Dist = (p X u).magnitude()
23. Compute the collision point relative to CM.
        Car1.collisionPoint = vertexList1[i] – car1.CMPosition
        Car2.collisionPoint = vertexList1[i] – car2.CMPosition
23. Compute the collision normal vector by taking the result of the cross product of u and p and crossing it with u.
        collisionNormal = ((u X p) X u).normalize()
24. Compute the velocity at the point of collision, which is the sum of the velocity of the CM and the tangential velocity.
        V1 = Car1.CMVelocity + Car1.angularVelocity X Car1.collisionPoint
        V2 = Car2.CMVelocity + Car2.angularVelocity X Car2.collisionPoint
25. Compute the relative velocity Vr = V1 – V2
26. Compute the relative normal velocity Vrn = dot product of Vr and collisionNormal
27. If 0 < proj.magnitude() <=edge.magnitude() and dist <= collision tolerance and Vrn < 0, then a collision occurs.
28. End vertex-edge loop.
```

28. If vertex-vertex check fails, then it goes on to check for penetration by iterating through each vertex in the list of the first car to see whether it lies within the bounds of the edge of another car.
29. Begin check for penetration loop.
30. Perform a standard point-in-polygon check using the vector dot product to determine whether any vertex of one polygon lies within the bounds of the other polygon.

        p = vertexList1[i] – edge vertex 1
        dot = dot product of p and edge

If dot is less than 0, it indicates a penetration.
31. End penetration loop.
32. If there is a collision or a penetration, then determine the linear and angular velocities of each car immediately after the impact using equation 10-1.

Figure 9-2. Fi

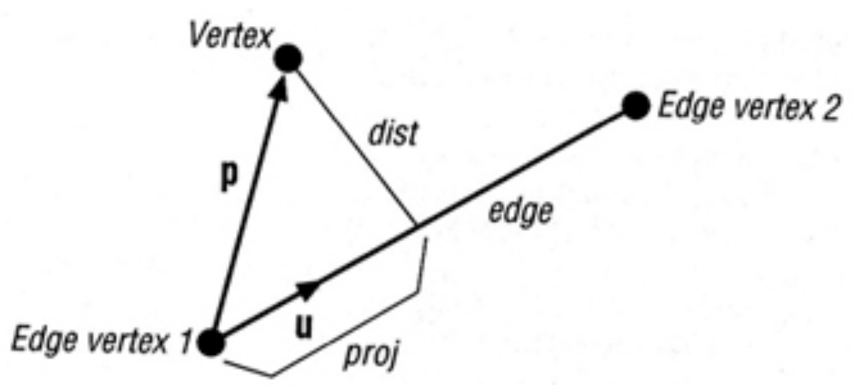


Figure 10-3. Vertex-Edge Check

Chapter 11: Car physics engine structure

The following figure shows the class diagram of the car physics engine describing the system's classes, their attributes, and operations. (Only some of the attributes and operation are listed)

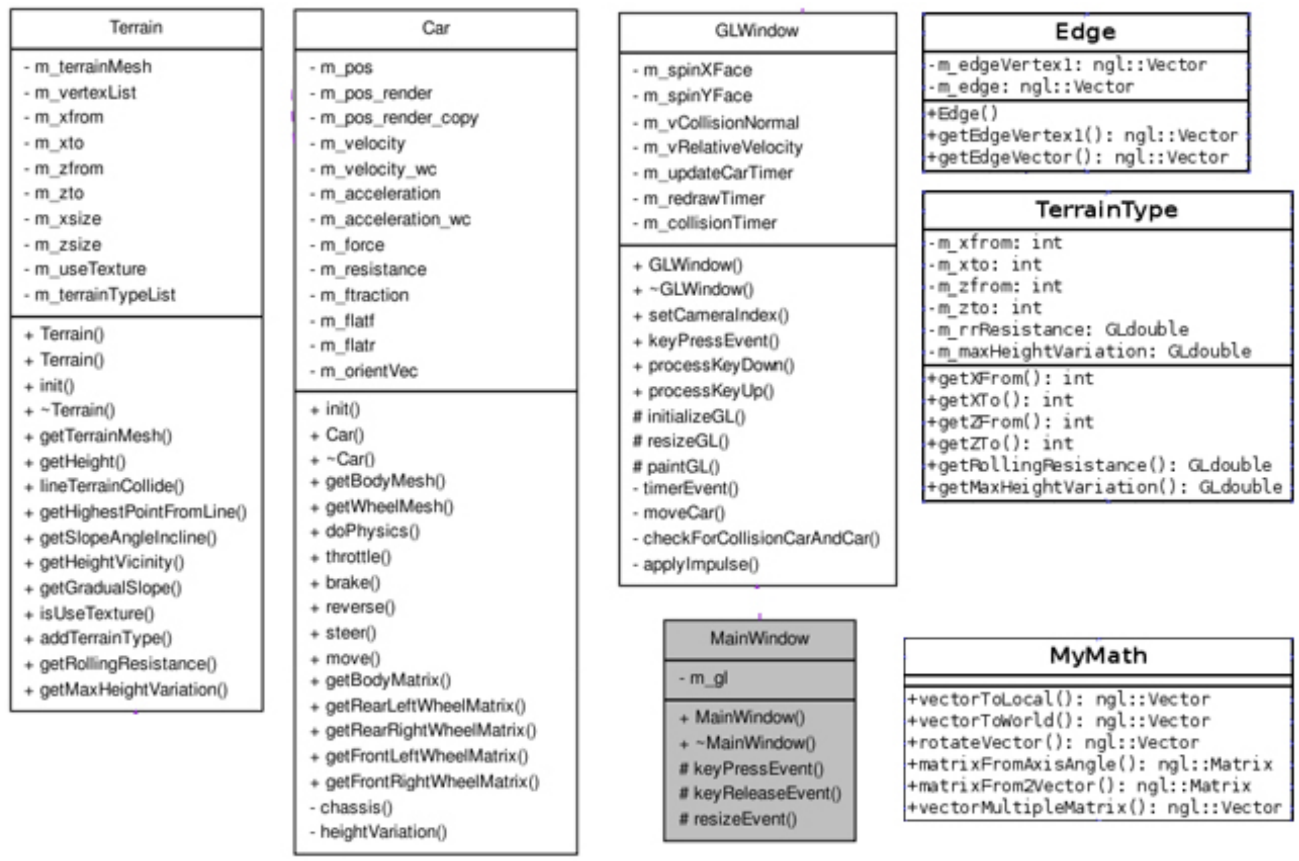
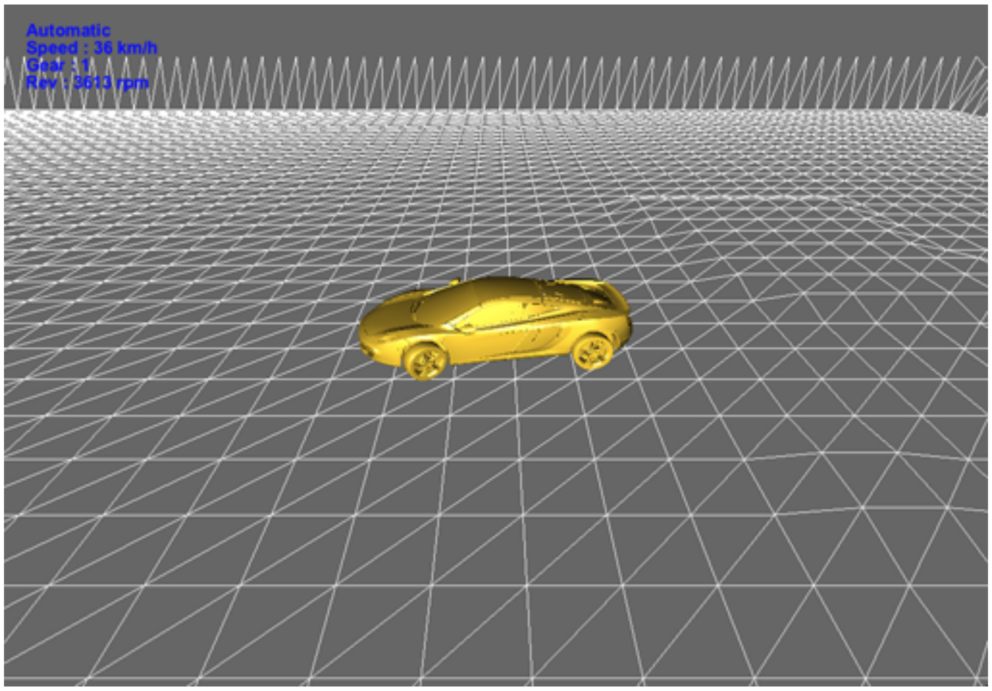


Figure 11-1. Class diagram

MainWindow is the main window, which contains a GLWindow widget. The GLWindow contains the drawing element, handles the collision response between two cars, and processes the keyboard input to control the cars. Car is the class that loads the car mesh, compute the car physics, and send the rotation matrices to the GLWindow to re-render the car. Terrain loads the model of the terrain, and calculates the height from a given position. It also contains an array of TerrainType objects, which define the properties of the terrains. Edge is the data structure use for vertex-edge collision detection. MyMath contains important function for mathematical calculations such as matrixFrom2Vector, which computes a rotation matrix require to rotate a source vector to a destination vector.

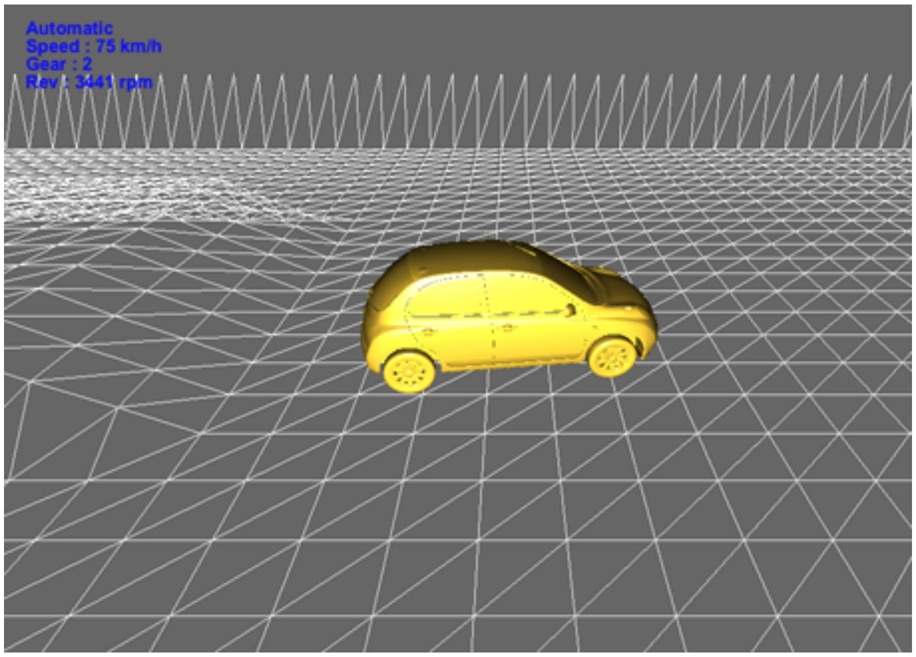
Chapter 12: Result and Analysis

To demonstrate the capability of the car physics engine, three different types of car (a sport car, a supermini, and a minibus) have been used to test the engine. Each car has been configured with different parameters such as mass, height of CM from the ground, car's size, cornering stiffness, suspension stiffness, and gear ratios. Driving the car gently and vigorously, the engine is able to simulate the physics of cornering at low speed, cornering at high speed, oversteering, understeering, and drifting pretty accurate. The suspension seems to look quite realistic and the user also feels the effect of weight transfer from handling the car. The speed of simulation is good even though the high-resolution models are being used.



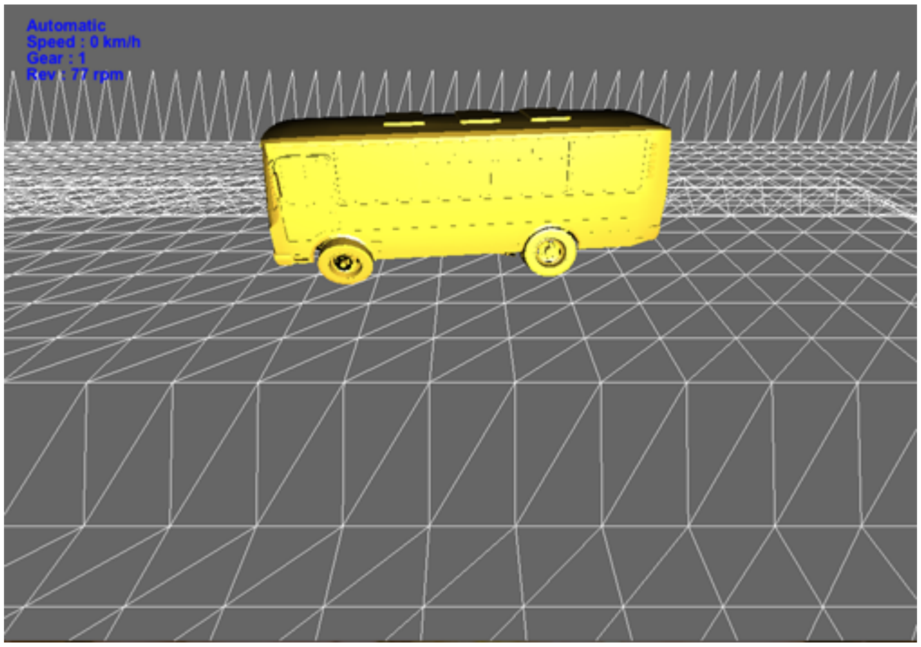
```
Automatic
Speed : 36 km/h
Gear : 1
Rev : 3613 rpm
```

Figure 12-1. McLaren MP4-12C



```
Automatic
Speed : 75 km/h
Gear : 2
Rev : 3441 rpm
```

Figure 12-2. Nissan Micra



```
Automatic
Speed : 0 km/h
Gear : 1
Rev : 77 rpm
```

Figure 12-3. A minibus

Chapter 13: Conclusion

A commercial grade car physics engine involves many areas of physics to achieve realistic results. This thesis only shows some techniques to simulate the dynamics of a car. While some calculations in the engine are only approximation and some do not rely on real physics, it produces good visual results and supports many types of car. The following is a list of known issues and limitations to the current system.

- With simple suspension system, the engine only supports cars with four wheels.

- The engine uses simple analytical methods rather than real physics to determine when will cars lift off the ground and how they land on the ground. Thus, it provides realistic result only to a certain extent.

- The collision response between two cars is not yet fully implement and the engine does not support collision response for multiple cars.

As mentioned earlier that car physics engine is a big topic. Some features can be improved and there are many more features that can be added to achieve more realism. Here is a list of some features that can be added.

- Improve the suspension, which can be achieved by adding more degrees of freedom for the chassis movement as well as adding dampers.

- Add different types of suspension such as McPherson, double wishbone, and dependent.

- Modify the engine to support motorcycles and cars with more than four wheels.

- Treat wheels as soft bodies to simulate the visual deformation of tires.

- Modify the transmission module to support different gear types, which are FWD, RWD, and 4WD.

- Add more properties for the terrain to support different surfaces such as asphalt, snow, ice, sand, grass, and earth.

- The engine should be improved to be more heavily based on real car physics rather than simple approximations but at the same time does not cut back on realism in the interest of fun.

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