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ORIGINAL ARTICLE

A fast hybrid computation model for rectum deformation

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Abstract It is a challenging task to realistically reproduce the complex deformation of soft bio-tissues in a surgical operation, especially when large deformations and movements exist. A hybrid model which we call the beads-on-string model is presented to handle the deformation and collision of the rectum in a virtual surgery simulation system. Specially tailored for this purpose, our model takes multiple layers to capture the dynamics of the rectum in an efficient manner. Inspired by the shape similarity between a rectum with regular bulges and a string of beads, we use a Cosserat rod model, coinciding with the centreline of the rectum, to calculate its deformation subject to external forces. We introduce rigid spheres, analogy to beads, moving along with the rod to approximate the shape of the rectum in handling collision. In addition, the beads (rigid spheres) provide a natural interlayer to map the deformation of the centreline to the associated mesh which presents detailed geometry of the rectum. Our approach is carefully crafted to achieve high computational efficiency and its multi-layer structure is designed to reproduce the physics of the deformation of the rectum.

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1 Introduction

Colorectal cancer is a life threatening disease. It is the third most common cancer after breast and lung cancers with 37,514 new cases reported in the UK in 2006 alone [3]. Surgery remains the most common treatment option with 80% of patients undergoing an invasive procedure. Due to the complexity of the anatomy and the delicate structure of the bowel, removing the cancerous tissue fully and cleanly is a very skillful job. However, such skills are born of experience and the reality is many new doctors gain these skills by operating directly on patients, which presents both a risk to the patient and inevitably prolongs the training for capable surgeons [35].

A virtual reality based simulator allows trainee doctors to learn their techniques by operating on a virtual patient. There are a number of computer assisted surgery simulators available and their efficacy is well attested to. However, such simulators have primarily been for operations where there are only small movements and small deformations of soft tissues, such as knee surgery.

Bowel cancer surgery is very different: to be effective as a training tool, the graphical representation, the sensation of force feedback (haptic feedback) and the deformation response of the soft tissues need to be realistic. Accurate representation of these elements is incredibly complex. To make matters worse for simulation, during an operation the rectum may collide frequently with other surrounding organs or even with itself. This requires a collision-detection algorithm which can handle the collision robustly and detect the collision efficiently in accordance with the deformation of the rectal structure.

To meet the requirement of real-time performance and sufficient accuracy, we have developed a hybrid model which incorporates several functional layers to handle the complex physical phenomenon under an efficient and self-consistent framework. Our focus is on the visual representation of the motion and deformation of the rectum when being interacted with the surgical instruments, where the collision handling is naturally included as part of the deformation model. A Cosserat rod model is introduced to continually capture the large deformation of the colorectal structure with good accuracy. It parameterizes the centreline of the rectum with curved material coordinates, which are moving along with the deforming structure. Rigid spheres attach to the Cosserat rod, just like beads on a string. These spheres approximate the 3D shape as bounding volumes for collision detection and handling. The spheres also provide a useful interface to map the deformation of centreline (Cosserat rod) to a detailed mesh. The mapping operation is based on the control cage constructed by vertices from the spheres using mean value coordinate interpolation.

Combining efficiency with accuracy in a computation model for complex deformations fills a technical gap of existing virtual surgery simulators. Applications of virtual surgery simulation rely on the capability of handling large non-linear deformations in real time without compromising the image quality. This target is achieved with our framework by radical but rational simplifications, which cut off unnecessary computations. This framework maintains a consistent computational structure of multiple layers where the deformation and collision computation shares the same module with the beads layer. It allows significantly flexible anatomical body parts, such as the rectum, to be simulated with confidence.

This work is a continual development of our previous research [11, 13]. Here we will focus on the computational model for deformation simulation. The haptic feedback is out of the scope of discussion in this paper.

The remainder of this paper is structured as follows. Section 2 reviews the recent advances in the related field, Sect. 3 outlines the idea of the algorithm, Sect. 4 discusses the material properties of bio-tissues and the deformation mechanism of the Cosserat rod model, Sect. 5 introduces the collision-handling procedure, Sect. 6 implements the mean value coordinate interpolation for controlling the detailed mesh with the beads on a string, Sect. 7 gives out some results of the numerical tests and Sect. 8 concludes the paper.

2 Related work

Though it is still an extremely challenging task to construct a complete and faithful virtual organ that allows real-time and realistic visual and haptic rendering, many efforts have been taken to develop novel approaches in both hardware and software implementations to push the boundary of the technology for virtual surgical training and planning [1, 10, 16–18, 34]. The computation of the deformation of soft organs/tissues is one of the major elements in such surgical simulation tasks [10, 16, 17, 30, 34].

The mass spring system (MSS) [22, 36] and the finite element method (FEM) [6, 10, 43] are the most popular methods for soft tissue modelling. The MSS uses a network of mass points connected with springs to mimic the soft object, while the FEM numerically solves the governing partial differential equations using approximation of discretized elements. The FEM can provide results toward a satisfying accuracy but its computation is arguably resource demanding. The real-time deformation models tend to trade-off with the accuracy or to sacrifice the physical nature of the deformation in its computation to meet the requirement of real-time performance [29]. Other physically based approaches, such as the finite volume method [41], boundary element method [24] and the recently developed mesh-free approaches [15, 37], are introduced to model the deformation of soft objects, but they share similar drawback of being computationally intensive as the FEM, which restricts their usage in real-time applications like surgery simulation.

The rectum as a long slender object suggests that we are able to approximate it with its centreline [11, 23]. The centreline representative uses much fewer degrees of freedom than the volumetric tetrahedron representation or the surface mesh model. This proves beneficial for its efficiency. In practice, the Cosserat rod model provides a natural abstraction to describe the physics of deformation by adopting a moving coordinate framework along the centreline curve [4]. It has attracted attentions in graphical research [7–9, 40] due to its ability to model the complex behaviors of 1D entity with both good physical accuracy and elegant mathematical manner. Latter, this model was extended to approximate the dynamic behavior of a volumetric object with a construction of welded Cosserat rods [12].

It may be elegant and efficient to use a curve, i.e. the centreline, to capture the deformations of the whole structure of a long thin structure, but an additional layer is required to map such deformations back to the polygon mesh which holds the geometric data for rendering. The approach of skinning was used by some researchers [20, 38] to generate deformations of the mesh of an intestine based on its centreline. However, the classic smooth skinning method suffers from a number of anomalies, including the so-called "collapsing joints" (significant volume loss around a joint when the joint bends) and "candy-wrapper artefacts" (an obvious shrinkage similar to a candy wrapper when a joint twists) [14], which results in degeneration of the visual presentation of the mesh model. Although an adaptive sampling technique [20, 38] was introduced to solve the problem using more joints in areas of high curvature, it brings in additional computational costs associated with the insertion of new sampling nodes (joints). Li et al. [31] used the Laplacian method to deform the mesh consistently with the centreline, providing satisfactory deformed shapes; however, it is very difficult to achieve real-time performance. Instead of developing novel skinning approaches [20], we introduced the well established interpolation techniques of mean value coordinates [27, 28, 32] to generate the mesh deformation by providing surrounding cages. The weights of interpolation can be pre-computed for the fast extraction of the deformed surfaces. The vertices of the control cages are calculated in run-time according to the associated beads (rigid spheres) attached to the centreline.

To produce convincing visual and haptic feedback to the user, the collision of the rectum with other tissues, surgical tools and itself must be detected and handled efficiently. The collision detection for rigid objects has been well studied [42]; various bounding volume approaches are developed to check the interpenetration among objects using simple geometry that approximates the actual shape. Bounding spheres were used to construct the bounding deformation tree for collision detection with deformed objects [25]. An effective bounding spheres approach [20] is used in our model due to its simplicity and high efficiency. Moreover, the bounding spheres (beads) in our model not only approximate the bounding volume but also provide a useful interface for mapping deformations of the center curve to the 3D geometric mesh.

3 Overview of the algorithm

The beads-on-string model presented in this paper is to handle the deformation and collision of the rectum for the purposes of virtual surgery simulation. This idea was firstly inspired by the observation that a rectum with regular bulges looks similar to a string of beads.

In our model, as illustrated in Fig. 1, the deformation of centreline is controlled by the mechanical model of Cosserat

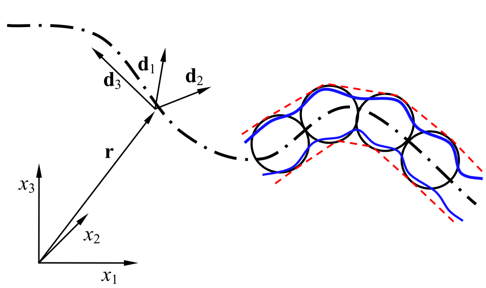


Fig. 1 Illustration of the beads-on-string model (without losing generality, a 2D diagram is given)

rod [4], and a string of beads (spheres in Fig. 1), which represent the approximated shape of the rectum, are rigidly attached to the centreline. The Cosserat rod model represents a rod as a space curve with a local coordinate frame attached to each point of the curve. It can be bended and twisted. A cage (the red dashed line in Fig. 1) is created by a selection of vertices on the bead spheres, which is surrounding the mesh representation of the real rectum. As the beads are moving and rotating, the shape of the cage changes accordingly. The shape change of the cage is then mapped to the deformation of the rectum mesh (the blue solid line in Fig. 1). A mean value coordinate interpolation scheme [27] is chosen to produce a smooth deformation of the rectum mesh.

The collision-handling model is consistent with our deformation model, while the beads are used to detect the collision as bounding spheres. The use of bounding volume in collision detection is not new and a similar approach was implemented in [20]. In our model, an additional layer of beads provides an interface to map the deformation of centreline to the associated mesh in a consistent manner where it presents a natural bounding volume approximation in collision handling.

4 Deformations of centreline—a 1D string

4.1 Material properties

The behavior of soft bio-tissues in response to the external forces is complicated. The experimental observation [21, 39] revealed the non-linear dependency between the force and displacement (or stress and strain). Noticeable change in the stress–strain curve between different loading cycles was obtained in cyclic loading test, and the viscoelastic behavior of the material was identified in stress-relaxation tests as well [39]. The anisotropic constitutive relations was reported in [21] revealing the difference of mechanical property along circumferential direction and meridional direction of in vivo porcine intestine.

Rosen et al. [39] described the constitutive relations of the soft tissue with the so-called EXP2 model, where the stress is a complex function formed by a linear part of the strain and an exponential part of the strain square. This model provides a good fit to the experimental data in their report. The EXP2 model is written as follows:

$$\sigma = \beta\left(e^{\alpha\varepsilon^2} - 1\right) + \gamma\varepsilon \tag{1}$$

where σ is the stress, ε is the strain and α, β, γ are the material parameters indicating the stiffness of the material.

The EXP2 model treats the material as a hyper-elastic one that the stress non-linearly depends only on the strain opting out the effect of viscoelasticity and the influence of

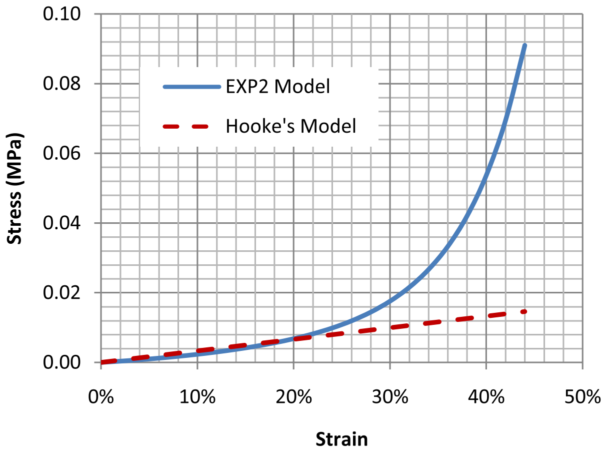


Fig. 2 Comparison of linear (Hooke's) and non-linear (EXP2) models

loading history. This model is proved to be complicated and inefficient in our computation as the deformation state must be determined through iterative calculations due to its non-linearity. Therefore, we use the linear elastic Hooke's law with constant stiffness instead. With this radical simplification, the bending stiffness and the torsion stiffness in the latter developed Cosserat model are constant and do not need to be updated at run-time. As shown in Fig. 2, major derivation between the two material models appears when the strain value is large (>25%). That implies that the linear model is capable of providing a fair approximation without losing much accuracy when the rectum in question is not subject to extremely large external forces. In practice, gentle gripping and pushing are applied to the soft tissues or organs during operation to avoid bleeding or other complications due to tissue damages, which means our simplification is unlikely to produce grossly inaccurate simulations.

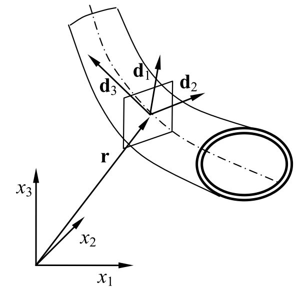
Using data from [39], we fit the Young's modulus of our linear model as 33.2 kPa. And we use a Poisson's ratio of 0.35, which indicates the compressibility of the material.

4.2 Cosserat rod model

In the Cosserat rod model, a long object is presented by a collection of points on a curve (the centreline of the rectum in our case) with a coordinate framework $\langle \mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3 \rangle$ attached to each point, where finding the centreline can be automated by the technique developed in [5]. The translation of the point presents the movement of the given cross section at that position and the rotation of the local coordinate frame presents the tilting and twisting of the cross section (see Fig. 3).

In practice, the director $\mathbf{d}_3$ of the local coordinate frame is chosen to be parallel to the tangent direction of the curve, and the other two directors are perpendicular to $\mathbf{d}_3$. Given

Fig. 3 Cosserat rod model



$\mathbf{r}(s)$ as a point on the selected curve, where s is the parametric coordinate of the curve length, we have

$$\mathbf{d}_3 \times \frac{\partial \mathbf{r}}{\partial s} = 0; \qquad \mathbf{d}_1 \times \mathbf{d}_2 = \mathbf{d}_3. \tag{2}$$

Using the definition of Darboux vector $\boldsymbol{\omega}$ as

$$\frac{\partial \mathbf{d}_i}{\partial s} = \boldsymbol{\omega} \times \mathbf{d}_i, \quad i = 1, 2, 3, \tag{3}$$

we can have the flexures (κ_1, κ_2) and the torsion (τ) of the rod as

$$\boldsymbol{\omega} = \kappa_1 \mathbf{d}_1 + \kappa_2 \mathbf{d}_2 + \tau \mathbf{d}_3 \tag{4}$$

where the flexure relates to the curvature of the curve describing the degree of bending of the rod and the torsion describes the twist of the rod along its tangent $\mathbf{d}_3$. The axial strain which denotes the length changes of the rod is defined as follows:

$$\varepsilon = \frac{\partial \mathbf{r}}{\partial s} \cdot \mathbf{d}_3 - 1. \tag{5}$$

The deformation of a Cosserat rod in question can be described with the above four variables: the flexures (κ_1, κ_2), the torsion (τ) and the axial strain ε. The shape of the center-curve can be resolved by integration of the four variables along its length.

The overall elastic potential consists of three parts, the bending, twisting and stretching potentials, which are written as follows:

$$\begin{aligned} U_b &= \frac{1}{2}\int B\left(\Delta\kappa_1^2 + \Delta\kappa_2^2\right) ds \\ U_t &= \frac{1}{2}\int T \Delta\tau^2 \, ds \\ U_s &= \frac{1}{2}\int EA\varepsilon^2 \, ds \end{aligned} \tag{6}$$

where $\Delta\kappa_1$ and $\Delta\kappa_2$ are the differences of the flexures with respect to the initial shape without deformation, $\Delta\tau$ is the difference of the torsion with respect to the initial shape, and ε is the axial strain. B is the bending stiffness, T is the torsion stiffness, E is the Young's modulus and A is the area of the cross section. $B = EI = \pi E R^3 h$, with area moment of inertia of the cross section, $I = \pi R^3 h$, where R is the radius of the rectum and h is the thickness of its wall. $T = GJ = 2\pi G R^3 h$, where $G = \frac{E}{2(1+\nu)}$ is the shear modulus and $J = 2\pi R^3 h$ is the polar moment of inertia. Here ν is the Poisson's ratio. As mentioned in the previous section, we use a linear elastic model with the material stiffness B, T and EA being constant.

To solve this problem numerically, the centreline curve as a Cosserat rod is discretized into N sections with $N+1$ nodes. The values of the flexures, torsion and axial strain at an arbitrary point can to be interpolated from the nodes' values. For example, the axial strain is given as follows:

$$\varepsilon(s) = \frac{s_{k+1} - s}{s_{k+1} - s_k}\varepsilon_k + \frac{s - s_k}{s_{k+1} - s_k}\varepsilon_{k+1}, \qquad s \in [s_k, s_{k+1}) \tag{7}$$

where s is the length parameter of the given point, s_k is the length parameter for the kth node and ε_k is the axial strain value at node k. Replacing the axial strain with the flexure or torsion respectively in (7), we can formulate these variables in a piecewise linear manner by the nodes' values.

From [12], the dynamics of a Cosserat rod can be given by solving the control equations which take the following form:

$$M\ddot{\mathbf{q}} + C\dot{\mathbf{q}} + K\mathbf{q} = \mathbf{R} \tag{8}$$

where $\mathbf{q}$ is the general strain vector assembled with the node values of the axial strain, flexures and torsion, $\ddot{\mathbf{q}}$ is the acceleration of $\mathbf{q}$, and $\dot{\mathbf{q}}$ is the velocity. M is the general mass matrix, C is the damp matrix and K is the stiffness matrix. $\mathbf{R}$ is the residual part. Chang et al. [11] and Li et al. [31] suggested solving the problem in a quasi-static fashion by dropping off the inertial term and the damping term, which largely simplifies the problem. Only the elastic term stands in the control equations leading to

$$K\mathbf{q} = \mathbf{R} \tag{9}$$

The control equations degenerate to a system of linear equations which can be solved much faster. Considering the motion of the rectum is usually during a surgical operation, we choose to model the problem with the simplified quasi-statistic model which can improve the computational performance significantly.

However, simply adopting the quasi-static model could result in a sudden flip of the centreline sometimes even when the change of the external force is smooth [31]. It may not be a big issue in other applications but this certainly decreases the quality of images we created for surgical simulation. Besides, the elastic model of (9) makes the rectum look stiff and unrealistic. It is expected that the rectum will settle down to a rest position under gravity once it is moved by the surgical instrument. With the elastic model, the shape tends to restore to its initial configuration once the external force disappears, which disagrees with the surgeon's expectation. Hence the non-linear viscous behavior of the rectum should be considered in order to provide convincing images.

It should be noted that the bending and twisting resistance of the rectum tends to be governed by the viscous law, while its axial extension is elastic. Therefore, we have to separate the general strain vector $\mathbf{q}$ into two groups: $\mathbf{q}_e$ stands for the axial strain values and $\mathbf{q}_v$ for viscous values of flexures and torsions. In recent development of [8], the viscous effect can be modelled in a formulation similar to the elastic one by replacing the strain term with the strain rate. Thus, we are able to include the viscous effect of the rectum by altering (9) as follows:

$$K\begin{Bmatrix} \mathbf{q}_e^t \\ \frac{1}{\Delta t}(\mathbf{q}_v^t - \mathbf{q}_v^{t-1}) \end{Bmatrix} = \mathbf{R}^t \tag{10}$$

where the superscript t means the value at current time t, $\mathbf{q}_v^{t-1}$ is the known value of viscous terms computed in previous time step, and Δt is the time-step size.

When a flexible rod is suspended horizontally by holding its two ends, it sags under gravity. In our simulation, when the rod is elastic, the deformed shape is approximated by a quartic polynomial; when the rod is viscous, the deformed shape is a catenary (see Fig. 4).

In order to correct the possible sudden flip of the Cosserat rod, the inertial force should be included in the computation. But directly solving (8) is not applicable in our application due to its computational complexity. This is because the general mass matrix is not diagonal and sparse and the computation of its inverse is by no means cheap [8]. To gain real-time performance and to maintain the smooth transition between frames by correcting the possible flip, we introduce a simplified approach instead. The idea is that we firstly estimate the motion of the Cosserat rod using (10) without considering the inertial force, and then we correct the estimated motion of each individual node separately as if the effect of the discretized mass at the node were included. Let us denote the estimated position of the kth node at time t as $\widehat{\mathbf{r}}_k^t$, which can be integrated from the general strain $\mathbf{q}^t$. We can write the corrected position $\mathbf{r}_k^t$ as follows:

$$\mathbf{r}_k^t = e^{-\xi\frac{m_k \Delta r}{\Delta t^2}}\widehat{\mathbf{r}}_k^t + \left(1 - e^{-\xi\frac{m_k \Delta r}{\Delta t^2}}\right)\mathbf{r}_k^{t-1} \tag{11}$$

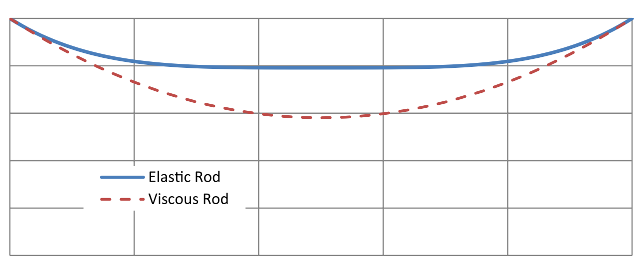


Fig. 4 Deformation of elastic and viscous rods under gravity

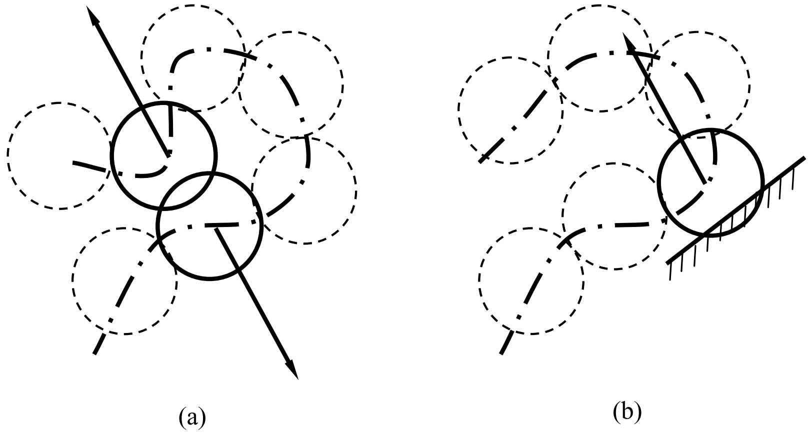


Fig. 5 Collision correction: (**a**) self-intersection; (**b**) collision with environment

where m_k is the discretized mass of the kth node, Δt is the time-step size, $\mathbf{r}_k^{t-1}$ is the position of the kth node at time $t-1$, ζ is a user-defined regulator for which we use value of $\frac{\pi}{16EA}$ with E as the Young's modulus and A as the area of cross section, and Δr is a measure of the deflection between estimated shape at time t and shape at time $t-1$, which is written as

$$\Delta r = \max_{k=1,2,\ldots,N+1} \left\| \left(\widehat{\mathbf{r}_k^t} - \mathbf{r}_k^{t-1} \right) \right\|. \tag{12}$$

In (11), the larger the value of the regulator ζ, the more influence the previous frame will give, therefore the smoother the transition between frames will be. Such correction is analogous to a first-order filter used in [2] to incorporate the dynamic effects of cloth simulation. However, unlike Aguiar's method of [2], the coefficient in our approach is given explicitly and there is no coupling among neighboring nodes, and hence the correction step adds little burden to the whole computation but can improve the final presentation with smooth transition between frames.

5 Bounding volume for collision—beads on a string

As outlined in Fig. 1, the beads are linked by a thread of the Cosserat rod, which provides an approximation of the shape of the rectum. The center of an arbitrary bead is chosen to be located at an appropriate node of the discretized Cosserat rod described in Sect. 4. Each bead moves with its attached node and its rotation is defined by the general strains on that node. For collision detection and handling, such beads can be treated as bounding spheres.

We number the spheres in sequence according to their position on the string (Cosserat rod). There is no need to include the test of two neighboring beads for collision as the internal axial force can separate them to some extent. If an intersection among two beads which are not neighbors is detected caused by the inconsistent motion, two separating forces (the arrows in Fig. 5) are exerted at the two nodes on the Cosserat rod where the beads are centred. When a bead intersects with the environment, not with the rectum itself, we only exert a separating force at the beads as we treat the environment as rigid objects. Both cases are outlined in Fig. 5. Such separating forces are included in the computation of deformation of the Cosserat rod in the next time step and hence correct the intersection progressively. The strength of the force is calculated by the following equation:

$$f = 50E\Delta d^2 \tag{13}$$

where E is the Young's modulus, and Δd is the intrusion depth of the intersection. Such penalty force method could introduce unwanted kinetic energy into the system and as a result may cause oscillation [19]; however, its fast performance is extremely beneficial to fast simulation. It is noticed that we treat the Cosserat rod as a viscous rod which provides a mechanism to dissipate the input energy and stabilizes the system somehow.

6 Mean value coordinate interpolation

To produce real-time deformation of the rectum in surgery simulation, one possible solution is to use the smooth skinning technique by placing a chain of joints along the centreline, where each joint locates at the center of a bead. Since the rectum seldom twists but can bend with a big angle between adjacent beads, the "candy-wrapper" effect is not a big concern in the simulation. However, the "collapse joint" artefact can be obvious and occur often with the rectum's movement as showed in Fig. 6(b). To preserve the volume during deformation, here we adopt a similar technique of the cage-based deformation from Ju et al. [28]. We build up a cage at the binding pose where the whole rectum is fully extended. The deformation of the rectum follows two steps: firstly, when the centreline of a Cosserat rod deforms, each vertex of the cage can be recomputed; then the deformed cage drives the deformation of the rectum surface.

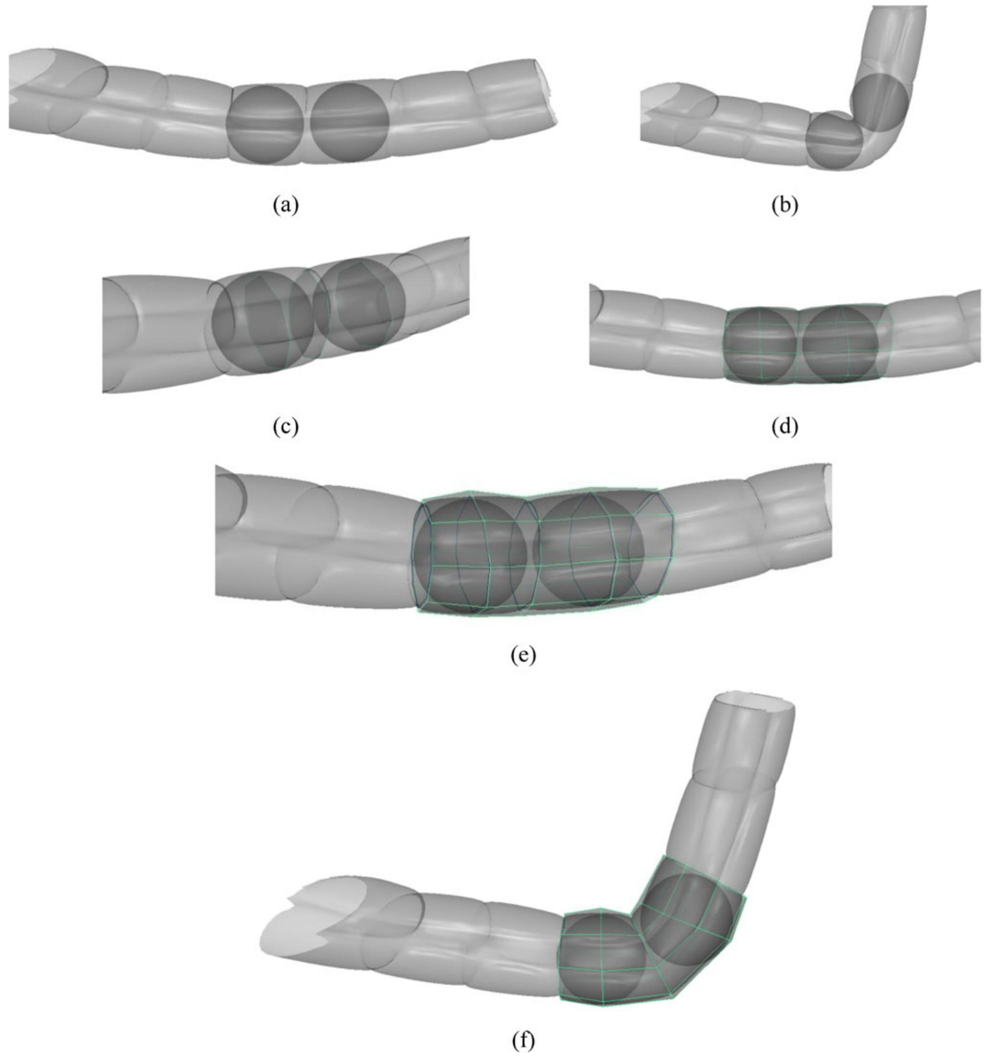


Fig. 6 Cage set-up and deformation. (**a**) Rectum model with beads; (**b**) Collapsing joint problem with smooth skinning; (**c**) Locating cross-sectional templates; (**d**) Initial tight cage; (**e**) Final cage; (**f**) Cage-based rectum deformation

6.1 Building cage cross sections

Differently from the complicated structure of a human body, for the rectum, only two types of cross sections (bead template and middle template) are used for the cage. The bead cross-sectional template is placed at the center of each bead, a ring of vertices being distributed evenly on the bead surface as showed in Fig. 6(c). The middle cross-sectional template is placed on the centreline between two adjacent beads. It holds the same number of vertices as the adjacent bead cross sections.

When the centreline deforms, the bead is relocated accordingly. The vertices on the bead cross section were fixed on the bead surface. For the vertices on the middle cross section, the new location is determined by the local frame at the middle point on the centreline between two beads. Since all the vertices on the cross section are deformed only by one local frame on the centreline, this scheme avoids the collapsing joint problem in the linear-blend skinning. Because the deformation of the cross section only involves a rotation around the centreline, all vertices on the template preserve the radial distance to the centreline. In the rectum simulation, because we do not have bulge features like the muscle deformation of a human character, we only include two types of cross sections in our template library.

6.2 Cage set-up

The cage is created at the rest pose where the whole rectum is fully extended. At each bead center, a cross-sectional template is located. The size of the template is tuned to attach each vertex on the bead surface. At each middle point between two adjacent beads, the middle template is located along the local frame of the centreline. Then each vertex has to be extended in the normal direction to be attached to the real rectum surface. Since the difference between the local frames of adjacent template is not very big, it is very easy to connect the corresponding vertices to form the embedding cage. To ensure proper cage-based deformation, the rectum mesh should be "embedded" inside the cage. However, the current cage is still intersected with the mesh as showed in Fig. 6(d). So for each vertex on all the templates, we extend it a small distance in the direction of the local normal direction to make sure the cage loosely surrounds the mesh (Fig. 6(e)).

6.3 Skinning

After the cage is set up, we can use the cage-based deformation techniques to deform the rectum surface. In the past few years, several works [26–28, 32, 33] have been published to deform a geometry embedded in a cage. Because of the heavy computational load of the harmonic coordinate method [26], it is not the best choice for the real-time requirement in the surgery simulation. For Green coordinates [33], since the deformed mesh may traverse the embedding cage, the collision-detection process will be unable to produce a stable result. Comparing the method of Positive Mean Value Coordinates (PMVC) [32] and Mean Value Coordinates (MVC) [27], since the rectum is bound to the cage in the fully extended shape, the cage at the rest pose is roughly convex, and negative values of the coordinates rarely appear in our practice. Therefore in our implementation, to achieve the highest efficiency, we adopt the Mean Value Coordinates to deform the rectum surface. Besides, considering the local effect of cage on the rectum surface deformation, when we compute the coordinates, we only consider the vertices on the adjacent templates. This localized optimization can further minimize our computation on cage deformation.

As the centreline is deformed from the Cosserat rod model, each bead will be transformed and reoriented. Since all the vertices on the cage are associated with the local frame of a certain point on the centreline, the new shape of the cage can be easily achieved, which in turn deforms the rectum mesh. The efficiency of the deformation strategy ensures the skinning process to be finished in real time. An added advantage is that the volume is largely preserved during bending due to the fact that only rotation operations are involved, as showed in Fig. 6(f).

7 Numerical tests

In order to test our deformation model and the collision-handling process, we simulate the movement of a piece of rectum. In Fig. 7, the rectum is released from a hanging position (Fig. 7(a)) and falls on the floor (Fig. 7, (b) and (c)). The geometric mesh of the rectum is modelled with reference to the real anatomic structure of human using Autodesk/Maya. From the test, we observed that the collision handling allows preventing visual penetration in an efficient manner for both self-collision and collision with the floor.

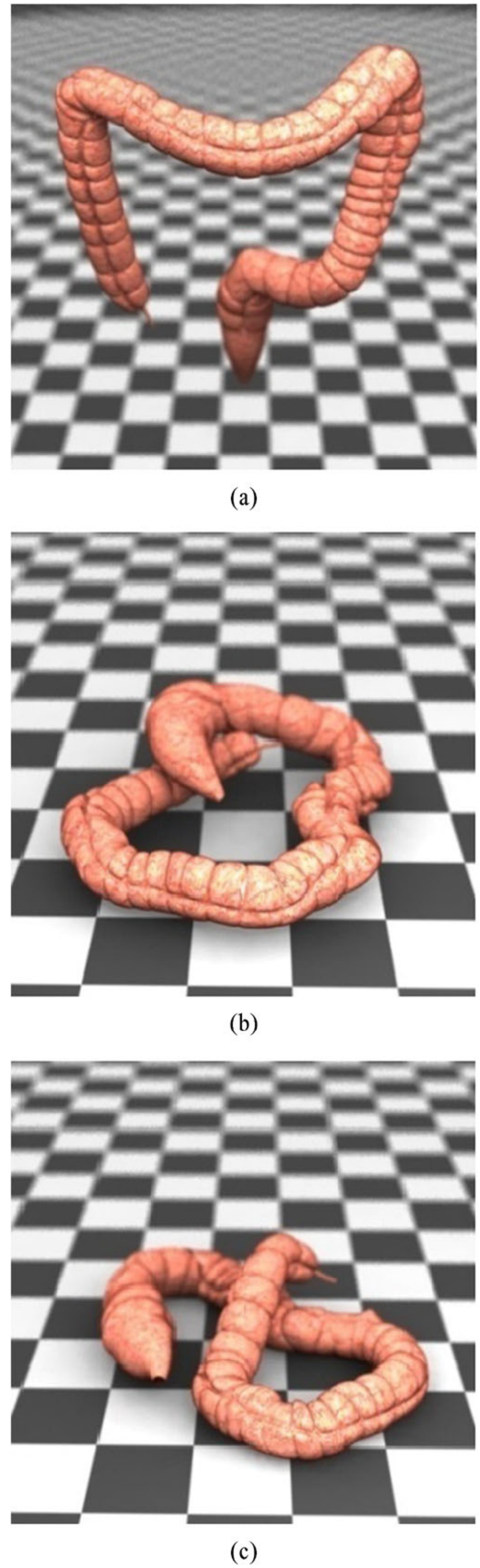
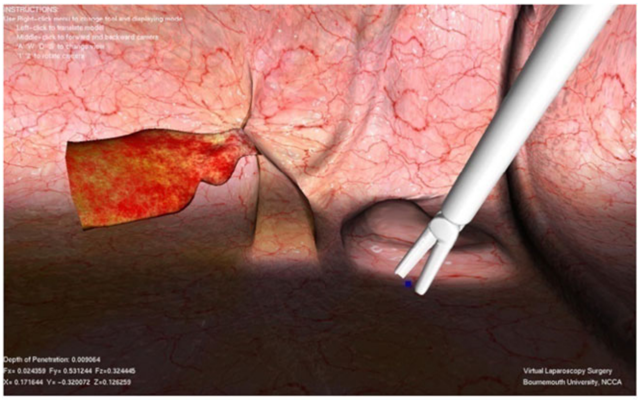
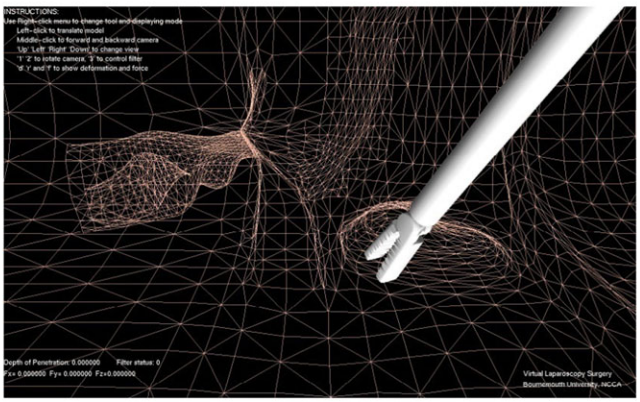


Fig. 7 Simulation of a rectum with our beads-on-string model: (**a**) initial hanging position (frame 0); (**b**) deformed shape of the rectum after falling on the floor (frame 60); and (**c**) deformed shape after settling (frame 270)



(a)



(b)

Fig. 8 Screenshot of the virtual reality system: (**a**) full rendered effects; (**b**) polygon mesh

167 nodes are used to produce the Cosserat rod deformation and 55 beads are used in collision handling. The deformation can be updated at a frame rate of about 80 fps. The deformation of the mesh is convincingly generated with the cage-based mapping mechanism. The PC we used is HP Workstation 4300, with Intel Pentium Dual CPU 3.2 GHz and 2 GB RAM, which is equipped with NVIDIA GeForce 7900 GTX video card.

To further validate our deformation method, we have incorporated it into a virtual reality platform for virtual surgery and have collected feedback from some surgeons specializing in colorectal surgery. The screenshot (Fig. 8(a)) shows the interface of the virtual reality simulation system, where the surgery instrument is controlled with a SensAble Omni haptic device. Figure 8(b) shows the mesh structure which defines the object's shape in operation. The surgeons have confirmed that the viscous model provides more convincing effects in terms of both the visual and haptic feedback than the elastic model. However, they also have pointed out that the force sometimes may experience unrealistic jiggling when the object has collided with several objects simultaneously. The condition can be improved by filtering the output force.

Our deformation model works well with the complex virtual surgery system. Its performance can satisfy the requirement for real-time applications, running at 40 fps with full rendering mode as shown in Fig. 8(a) and up to 60 fps for the wireframe mode as in Fig. 8(b). Although the force feedback in surgery simulation requires much higher updating frequency of nearly 1000 Hz, this has to be achieved through an additional interpolation filter in the haptic pipeline, which is out of the scope of our discussion in this paper.

8 Conclusions

We have presented a novel deformation method, known as the beads-on-string model, for significantly flexible and long thin soft tissues. In addition to other similar soft objects, this model is particularly effective for the simulation of the deformation of intestines/rectums. The structure of the method makes it straightforward for detecting and handling collision, which helps improve the computational efficiency. Apart from the physical accuracy, one major advantage is its real-time performance suitable for many real-time applications such as virtual surgery simulation. This is achieved with several effective means presented in the paper: a bounding sphere structure (beads) is used to approximate the shape for collision detection and handling; such beads are controlled with a mechanical curve, Cosserat rod, which is capable of capturing the dynamics of a long thin object under bending, twisting and axial pulling; the deformation is treated as a quasi-static problem, which brings further savings of computation; the deformation of beads and string is finally used to drive the deformation of the detailed mesh using mean value coordinate interpolation. This beads-on-string model can be solved efficiently with a numerical approach, where the Cosserat rod is discretized into several dozens of nodes depending on the complexity of the problem.

Unlike the Cosserat rod model in existing literature, which was developed to handle pure elastic deformations, our implementation includes the viscous deformation effect of bending and twisting which enhances the overall level of realism for both visual and haptic outputs. Replacing elastic mode with viscous mode is achieved by computing with the velocities of the related general strains. A non-linear filtering process is introduced to correct the possible flipping problem in the quasi-static model, producing smooth transitions between frames. The filtering process takes into consideration the inertial force using a rough estimation and enhances the animation quality.

Though the image quality and the simulation accuracy of our implementation are not as good as the off-line simulation programs, it has been specially tailored for real-time applications such as virtual surgery simulation. The test has shown that our implementation can provide a satisfying frame updating rate and good visual results. The feedback from the surgeons has confirmed that our model satisfies the needs for producing convincing deformations for the rectum in real time. However, the collision detection and handling approaches could be improved by considering more complex cases when multiple collisions coexist simultaneously at a local contact. More sophisticated collision mechanisms may be beneficial. This will be investigated in the future.

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References

1. Abate, A.F., Acampora, G., Loia, V., Ricciardi, S., Vasilakos, A.V.: A pervasive visual-haptic framework for virtual delivery training. IEEE Trans. Inf. Technol. Biomed. **14**(2), 326–334 (2010)
2. Aguiar, E., Sigal, L., Treuille, A., Hodgins, J.K.: Stable spaces for real-time clothing. ACM Trans. Graph. **29**(4), 1–9 (2010)
3. Anonymous: Latest UK cancer incidence and mortality summary—Numbers, July, 2009, Cancer Research UK. http://publications.cancerresearchuk.org/WebRoot/crukstoredb/CRUK_PDFs/mortality/IncidenceMortalitySummary.pdf (2009), Accessed 5 Aug., 2010
4. Antman, S.: Nonlinear Problems of Elasticity. Springer, New York (1995)
5. Baran, I., Popović, J.: Automatic rigging and animation of 3D characters. In: SIGGRAPH'07, Article 72 (2007)
6. Bathe, K.J.: Finite Element Procedures, 2nd edn. Prentice-Hall, London (1996)
7. Bergou, M., Wardetzky, M., Robinson, S., Audoly, B., Grinspun, E.: Discrete elastic rods. In: SIGGRAPH'08, Article 63 (2008)
8. Bergou, M., Audoly, B., Vouga, E., Wardetzky, M., Grinspun, E.: Discrete viscous threads. ACM Trans. Graph. **29**(4), 1–10 (2010)
9. Bertails, F., Audoly, B., Cani, M., Querleux, B., Leroy, F., Lévêque, J.: Super-helices for predicting the dynamics of natural hair. In: SIGGRAPH'06, pp. 1180–1187 (2006)
10. Bro-nielsen, M.: Finite element modeling in surgery simulation. Proc. IEEE **86**(3), 490–503 (1998)
11. Chang, J., Pan, J., Zhang, J.J.: Modelling rod-like flexible biological tissues for medical training. In: Magnenat-Thalmann, N. (ed.) 3DPH'09. LNCS, vol. 5903, pp. 51–61. Springer, Berlin (2009)
12. Chang, J., Shepherd, D., Zhang, J.J.: Cosserat-beam-based dynamic response modelling. Comput. Animat. Virtual Worlds **18**(4–5), 429–436 (2007)
13. Chang, J., Yang, X., Pan, J., Li, W., Zhang, J.J.: Beads-on-string model for virtual rectum surgery simulation. In: 3DAHSS, Chania, Greece (2010)
14. Chang, J., Yang, X., Zhang, J.J.: Continuous skeleton-driven skinning—a general approach for modelling skin deformation. Int. J. Image Graphics **9**(4), 591–608 (2009)
15. Chang, J., Zhang, J.J.: Mesh-free deformations. Comput. Animat. Virtual Worlds **15**(3–4), 211–217 (2004)
16. Comas, O., Taylor, Z.A., Allard, J., Ourselin, S., Cotin, S., Passenger, J.: Efficient nonlinear FEM for soft tissue modelling and its GPU implementation within the open source framework SOFA. In: Bello, F., Edwards, P. (eds.) ISBMS'08. LNCS, vol. 5104, pp. 28–39. Springer, Berlin (2008)
17. Cotin, S., Delingette, H., Ayache, N.: Real-time elastic deformations of soft tissues for surgery simulation. IEEE Trans. Vis. Comput. Graph. **5**(1), 62–73 (1999)
18. Delingette, H., Ayache, N.: Hepatic surgery simulation. Commun. ACM **48**(2), 31–36 (2005)
19. Drumwright, E.: A fast and stable penalty method for rigid body simulation. IEEE Trans. Vis. Comput. Graph. **14**(1), 231–240 (2008)
20. France, L., Lenoir, J., Angelidis, A., Meseure, P., Cani, M., Faure, F., Chaillou, C.: A layered model of a virtual human intestine for surgery simulation. Med. Image Anal. **9**(2), 123–132 (2005)
21. Hoeg, H.D., Slatkin, A.B., Burdick, J.W., Grundfest, W.S.: Biomechanical modeling of the small intestine as required for the design and operation of a robotic endoscope. In: ICRA'00, pp. 1599–1606 (2000)
22. Hong, M., Jung, S., Choi, M., Welch, S.W.: Fast volume preservation for a Mass–Spring system. IEEE Comput. Graph. Appl. **26**(5), 83–91 (2006)
23. Huang, P., Gu, L., Zhang, S.: Real-time simulation for global deformation of soft-tissue using deformable centreline and medial representation. In: Harders, M., Székely, G. (eds.) ISBMS 2006. LNCS, vol. 4072, pp. 67–74 (2006)
24. James, D.L., Pai, D.K.: Multiresolution Green's function methods for interactive simulation of large-scale elastostatic objects. ACM Trans. Graph. **22**(1), 47–82 (2003)
25. James, D.L., Pai, D.K.: BD-tree: output-sensitive collision detection for reduced deformable models. In: SIGGRAPH'04, pp. 393–398 (2004)
26. Joshi, P., Meyer, M., DeRose, T., Green, B., Sanocki, T.: Harmonic coordinates for character articulation. In: SIGGRAPH'07, Article 71 (2007)
27. Ju, T., Schaefer, S., Warren, J.: Mean value coordinates for closed triangular meshes. ACM Trans. Graph. **24**(3), 561–566 (2005)
28. Ju, T., Zhou, Q., van de Panne, M., Cohen-Or, D., Neumann, U.: Reusable skinning templates using cage-based deformations. ACM Trans. Graph. **27**(5), 1–10 (2008)
29. Kim, J., Choi, C., De, S., Srinivason, M.A.: Virtual surgery simulation for medical training using multi-resolution organ models. Int. J. Med. Robot. Comput. Assist. Surg. **3**, 149–158 (2007)
30. Kim, Y., Lee, K., Kim, W.: 3D virtual simulator for breast plastic surgery. Comput. Animat. Virtual Worlds **19**(3–4), 515–526 (2008)
31. Li, H., Leow, W.K., Chiu, I.: Elastic tubes: modeling elastic deformation of hollow tubes. Comput. Graphics Forum (2010). http://dx.doi.org/10.1111/j.14678659.2010.01647.x
32. Lipman, Y., Kopf, J., Cohen-Or, D., Levin, D.: GPU-assisted positive mean value coordinates for mesh deformations. In: SGP 2007, Aire-la-Ville, Switzerland. ACM International Conference Proceeding Series, vol. 257, pp. 117–123 (2007)
33. Lipman, Y., Levin, D., Cohen-Or, D.: Green coordinates. ACM Trans. Graph. **27**(3), 1–10 (2008)
34. Marchal, M., Allard, J., Duriez, C., Cotin, S.: Towards a framework for assessing deformable models in medical simulation. In: Bello, F., Edwards, E. (eds.) ISBMS 2008. LNCS, vol. 5104, pp. 176–184. Springer, Heidelberg (2008)
35. Molinas, C.R., Win, G.D., Ritter, O., Keckstein, J., Miserez, M., Campo, R.: Feasibility and construct validity of a novel laparoscopic skills testing and training model. Gynecol. Surg. **5**, 281–290 (2008)
36. Mollemans, W., Schutyser, F., Cleynenbreugel, J., Suetens, P.: Fast soft tissue deformation with tetrahedral mass spring model for maxillofacial surgery planning systems. In: Barillot, C., Haynor, D., Hellier, P. (eds.) MICCAI 2004. LNCS, vol. 3217, pp. 371–379. Springer, Berlin (2004)
37. Müller, M., Keiser, R., Nealen, A., Pauly, M., Gross, M., Alexa, M.: Point based animation of elastic, plastic and melting objects. In: SCA'04, pp. 141–151 (2004)
38. Raghupathi, L., Grisoni, L., Faure, F., Marchal, D., Cani, M., Chaillou, C.: An intestinal surgery simulator: real-time collision

processing and visualization. IEEE Trans. Vis. Comput. Graph. **10**(6), 708–718 (2004)
39. Rosen, J., Brown, J.D., De, S., Sinanan, M., Hannaford, B.: Biomechanical properties of abdominal organs in vivo and postmortem under compression loads. J. Biomech. Eng. **130**(2), 021020 (2008)
40. Spillmann, J., Teschner, M.: CoRdE: Cosserat rod elements for the dynamic simulation of one-dimensional elastic objects. In: SCA'07, pp. 63–72 (2007)
41. Teran, J., Blemker, S., Hing, V.N., Fedkiw, R.: Finite volume methods for the simulation of skeletal muscle. In: SCA'03, Aire-la-Ville, Switzerland, pp. 68–74 (2003)
42. Van den Bergen, G.: Collision Detection in Interactive 3D Environments. Morgan Kaufmann, Oxford (2003)
43. Wu, W., Heng, P.A.: A hybrid condensed finite element model with GPU acceleration for interactive 3D soft tissue cutting. Comput. Animat. Virtual Worlds **15**(3–4), 219–227 (2004)

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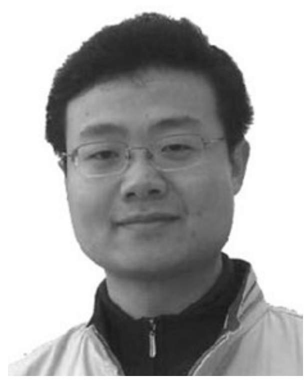

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